QUALITY CONTROL AND RELIABILITY

TECHNICAL REPORT TR 3

SAMPLING PROCEDURES AND TABLES FOR LIFE AND RELIABILITY TESTING BASED ON THE WEIBULL DISTRIBUTION

(MEAN LIFE CRITERION)

30 SEPTEMBER 1961
OFFICE OF THE ASSISTANT SECRETARY OF DEFENSE
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Quality Control and Reliability

The content of this technical report was prepared on behalf of the Office of the Assistant Secretary of Defense (Installations and Logistics) by Professors Henry P. Goode and John H. K. Kao of Cornell University through the cooperation of the Office of Naval Research. It was developed to meet a growing need for the use of mathematically sound sampling plans for life and reliability testing where the Weibull Distribution adequately approximates the test data.

FOREWORD

This technical report presents a proposed acceptance-sampling procedure together with tables of sampling plans for life and reliability testing when the underlying life distribution of items is of the Weibull form. The study upon which this report is based was done under a Cornell University contract sponsored by the Office of Naval Research. Part of the material was presented at the Seventh National Symposium on Reliability and Quality Control and is published in the Proceedings[1] of this symposium. Another part of the report was presented at the 1961 Annual Convention of the American Society for Quality Control and was published in the Transactions[2] of that convention.

The procedures and plans are for use when inspection of the sample items is by attributes with the life test truncated at some specified time. Lot quality is evaluated in terms of mean item life.

A set of conversion factors has been provided from which attribute sampling-inspection plans of any desired form may be designed for the Weibull model or from which the operating characteristics of any given plan may be determined. In addition, a comprehensive collection of Weibull sampling-inspection plans has been designed. Also included are tables of ratios for adapting the Military Standard 105B[3] plans to life testing and reliability applications. In all three of these elements of the study and the report, the exponential model has been included as a special case of the Weibull distribution. Both the procedure and plans are for product for which the value for the Weibull shape parameter is known or can be assumed. Conversion factors and tables are provided for a wide range of values for this parameter.

These proposed techniques represent generalizations of procedures and plans developed for the exponential case by Sobel and Tischendorf[4] and by Epstein[5]. Related work has been done by Gupta and Groll[6] for the gamma form of life-length distribution. Recently, an interim handbook of sampling procedures and tables based on the exponential distribution has been published by the Office of the Assistant Secretary of Defense (Installations and Logistics) based in large part on Professor Epstein's work[7]. It is hoped that the plans and procedures provided in this report will serve as a useful supplement to this work.

Henry P. Goode
John H. K. Kao
Department of Industrial and Engineering Administration
Sibley School of Mechanical Engineering
Cornell University, Ithaca, New York

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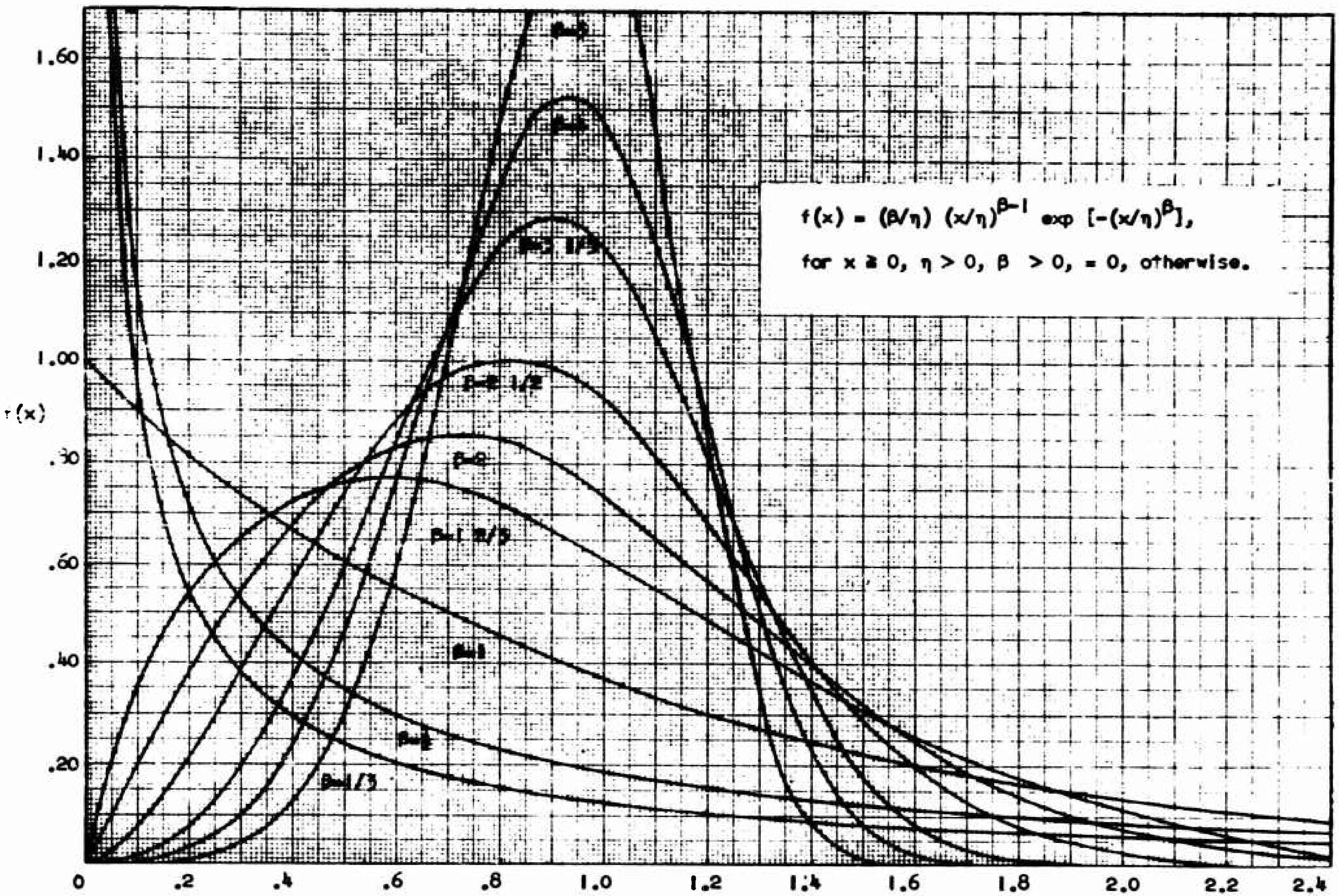

Figure 1. Plot of the Weibull Probability Density Function for Various Values of β - (η=1)

SECTION 1
INTRODUCTION

1.1 The Weibull Model in Life and Reliability Testing.

The Weibull distribution is a statistical model that has been found to be realistic for application in life-length and reliability testing for many mechanical and electronic components. It is a three-parameter model that takes the form of a general failure distribution in which the failure rate need not necessarily be constant.

Further discussion of this distribution as a life or failure model will be found in Appendix A of this report. Reference may also be made to a paper by Kao[8]. The exponential distribution, which is widely used as a statistical model in reliability analysis and inspection, can be considered as a special form of the Weibull, one which requires the assumption of a constant failure rate.

The three Weibull parameters are (1) a "scale" parameter, which is commonly symbolized by the letter α, (2) a shape parameter, for which β is the symbol commonly used, and (3) a location parameter, designated by γ. The general nature of the Weibull model may be observed from the density functions plotted in Figure 1. For use of the procedures and plans covered by this report, only values for the shape parameter, β, and for the location parameter, γ, need be determined; the value for the "scale" parameter, α, need not be known.

From Figure 1 it will be noted that the form of the distribution depends on the shape parameter, β. In this figure the Weibull probability density function has been plotted for various values of β. A plot for $\beta = 1$, the exponential case, has been included for reference. This initial set of Weibull sampling plans is for product for which the value of this parameter is known or can be assumed to approximate some given value. Conversion tables and sampling plans are provided for nine values for β ranging from 1/3 to 5.

A relatively small number but a broad range of values for β was selected for this initial study. The principal objectives were to develop practical methods and techniques and to explore the effect of differences in value for this parameter (which is the key one for the Weibull distribution). As the use of this distribution as a statistical model increases, additional conversion tables and sampling inspection plans may be constructed for intervening values for β, particularly in the widely encountered region ranging from 1/2 to 2.

In many applications the shape parameter, β, may be known for the product in question. From past analysis of life testing results, it may be established that some value of known magnitude may be expected regularly and so may be used in sampling inspection procedures. For example, for a certain class of electron tubes of receiving type, Kao[9] has found from study of approximately 2,000 tubes of a variety of types and applications that a value of 1.7 may be appropriate. For ball bearings, Lieblein and Zelen[10] found a mean value of 1.51 with 50% of approximately 5,000 bearings tested having β in the interval 1.17 to 1.74.

For products for which the value for β is not known, this parameter must be estimated using failure data from past inspection and research. A graphical method for estimation will be found in Appendix B of this report. Also, a simple and practical mathematical procedure is currently being developed and should be available soon[11]. Further information on estimation may also be found in papers by Kao[12,13].

Also, as indicated, in application of these plans and procedures, the value for γ, the location parameter, must likewise be known or assumed to approximate some given value. For many applications (for products for which there is risk of failure immediately after manufacture or after being put to use), a value of zero for γ can usually be assumed. Methods for estimation of γ when estimates must be made will be found in the appendix and in the references cited above.

1.2 Form of Acceptance Criteria

For the sampling-inspection plans and procedures presented in this technical report, lot or product quality is evaluated in terms of mean item life, μ. Work is under way on the development of related plans and procedures for which the evaluation will be for failure rate and for reliable life as criteria.

Inspection of sample items is by attributes; all that must be done in testing is to note the number of items that have failed and to do so without reference to the specific length of life for each. Testing of the sample items is stopped or truncated at the end of some preassigned test period length, t.

Specifically, the following acceptance-sampling procedure for life testing has been assumed:

1. Select at random a sample of n items from the lot.
2. Place the sample items on life test for some preassigned period of t time units.
3. Denote by y the number of failures observed prior to time t.

4. Accept the lot if $y \leq c$, a specified acceptance number; if $y > c$, reject the lot.

Curtailed inspection for submitted lots prior to t is possible for the rejection of the lot since it is possible to observe $(c + 1)$ failures before time t.

Note that this acceptance procedure is the same as that specified for the MIL-STD-105B sampling plans and other collections of attribute plans with the exception of the introduction of a testing truncation time, t. It is also possible (as for the 105B plans) to employ double or multiple sampling instead of single sampling as described above and by so doing reduce the average number of items at $p' \cong$ AQL that must be put on life test. However, the "economy" achieved is at the expense of longer elapsed testing time.

SECTION 2

THE BASIC CONVERSION FACTORS

2.1 The Function of the Factors.

The probability of acceptance for a lot, $P(A)$, under plans of the above form depends on the probability, p', of item life being less than (or equal to) the test truncation time, t. For cases for which β is known and with time, t, preassigned, p' is thus a function of mean item life, μ, only. The operating characteristics of any specified plan thus depend only on t and μ. In order to provide tables for general use in the design or evaluation of plans for any application rather than working in terms of specific values for t and μ, the dimensionless ratio t/μ is used. In application of the plans or tables to any application, a conversion between the ratio and specific t and μ values is extremely easy to make.

A set of conversion tables has been computed to provide for the Weibull distribution the connection between the dimensionless quantity t/μ and p' (Tables 1 and 2). With these tables, acceptance-sampling plans of desired form can be designed or evaluated using attribute sampling theories and practice.

For cases for which the lot size, N, is large in relation to the sample size, n, the number of failures prior to t approximates the binomial distribution with parameters n and p', where p' is defined as the area under the life-length distribution up to t. The probability of acceptance $P(A)$ depends on the cumulative number of failures prior to time t. This probability is given by

$$P(A) = P(y \leq c) = \sum_{y=0}^{c} \binom{n}{y} p'^{y} (1-p')^{n-y} \qquad (1)$$

The binomial distribution has been used for the sampling plans given in this report except for cases for which the sample size is relatively large. For these, the Poisson distribution has been used as an approximation to the binomial. The probability of acceptance for the Poisson is given by

$$P(A) = P(y \leq c) = \sum_{y=0}^{c} \frac{(np')^{y}}{y!} e^{-np'} \qquad (2)$$

An important use for the conversion tables provided in this report is in the adaptation of the MIL-STD-105B plans to reliability and life-testing applications. In describing the operating characteristics of these plans, the quality

of submitted lots is measured in terms of p', the per cent defective. With the conversion factors this form of description may be converted directly to measurement in terms of the t/μ ratio. With this conversion the 105B plans may be cataloged for appropriate choice in reliability applications. Alternatively, if some 105B plan has been selected, its operating characteristic curve may be determined in terms of the t/μ ratio, or if the testing time, t, has been specified, in terms of the lot mean, μ. An example employing such a conversion is shown later in this report. It should also be noted that with the matching plans provided in the 105B collection, the options of double-sampling and multiple-sampling are also available. The sample sizes and acceptance numbers listed may be used and the established procedures for employing this form of sampling in attribute inspection may be followed. Special tables of conversion ratios, together with procedures for their use, for directly applying the MIL-STD-105B plans to reliability and life-testing are included in Section 5 of this report.

2.2 Computation of the Conversion Tables.

The probability p', of an item failing prior to some testing time t is the value of the cumulative density function at t. For the Weibull model assumed in this report [Equation (A16)], this probability is,

$$p' = 1 - \exp\left[-(t/\eta)^{\beta}\right] \tag{3}$$

Since the mean of this Weibull distribution is, $\mu = \eta\,\Gamma(\frac{1}{\beta} + 1)$, p' may be re-written as,

$$p' = 1 - \exp\left(- \left[\frac{t}{\mu}\,\Gamma\left(\frac{1}{\beta} + 1 \right) \right]^{\beta}\right) \tag{4}$$

Solving for t/μ ,

$$t/\mu = \left[-\ln(1-p') \right]^{1/\beta} / \,\Gamma\left(\frac{1}{\beta} + 1\right) \tag{5}$$

This equation establishes the relationship between the dimensionless ratio t/μ and p', the probability of item life being equal to or less than t.

It may be noted that for the attribute form of sampling inspection considered here, only this dimensionless ratio between test time, t, and item mean life, μ, need be of concern. The Weibull scale parameter, η has been eliminated. In the mathematics of these plans and procedures it has been assumed that the Weibull location parameter, γ, has a value of 0 [see Equation (A16)]. If in application, however, γ has some non-zero value, all that is necessary is

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to subtract the value for γ from the value for t to get t_o, and from the true lot mean μ, to get μ_o. These converted values, t_o and μ_o, are then used for all computations. Any solutions in terms of t_o or μ_o can be readily converted back to real values by simply adding the value for γ. This procedure for handling the location parameter will be illustrated later in Example 3 and in Example 9. Of course, when $\gamma = 0$, only the parameter β (or b, which is $1/\beta$) must be known.

To put this relationship equation [Equation (5)] in a form for which numerical values for relationships may be more easily computed, the following change is made:

$$t/\mu = [-\ln(1-p')]^b / \Gamma(b+1) = \exp\{ \ln [-\ln(1-p')]^b \} / \Gamma(b+1)$$

$$= \exp\{ b \ln [-\ln(1-p')]\} / \Gamma(b+1) \ . \qquad (6)$$

Values for the expression

$$\ln [-\ln (1 - p')] \qquad (7)$$

were obtained from a table of the inverse of the cumulative probability function of extremes prepared by the National Bureau of Standards[14]. This table tabulates the function

$$y = -\ln (-\ln \Phi_y) \ . \qquad (8)$$

By substituting $(1 - p')$ for Φ_y the negative value of Expression (7) is obtained. Values for e raised to this power were read from the National Bureau of Standards tables of the exponential function[15]. Values for the gamma function, $\Gamma(b + 1)$, were obtained from a table prepared by Dwight[16].

A table of values for the per cent truncation, $(t/\mu) \times 100\%$, for various values of p' has been prepared. It is presented as Table 1. Values for p' range from .010% to 80% with the tabulated values selected in accordance with a standard preferred number series. For convenience in both tabulation and use, both the ratio t/μ and p' are expressed as percentages rather than decimal ratios.

For determining without interpolation the value for p' when some rounded value for the $(t/\mu) \times 100$ ratio is given, the relatively simple task of preparing a table of p' has been carried out.

The table of values for p' for various values for $(t/\mu) \times 100$ is presented as Table 2. Values for $(t/\mu) \times 100$ range from .010 to 100. Again, the values used for tabulation form a preferred number series. With this alternate table

available together with the basic original one (Table 1), a conversion may readily be made either way--from $(t/\mu) \times 100$ to p' or from p' to $(t/\mu) \times 100$. Also, it will be noted that the two supplement each other in that β values giving a compressed range of figures in one table give an expanded range in the other. This allows for somewhat more precise interpolation in conversion. The two together supply basic data for the design or evaluation of any life-testing and reliability sampling inspection plan based on the Weibull (or exponential) distribution and of an attribute form. For general information, the relationship between the $(t/\mu) \times 100$ percentage and p' as given by these two tables has been plotted in Figure 2 for each of the various β values.

2.3 Examples of Application.

Example (1)

One form of application that should be of considerable use is that of evaluating the quality protection afforded by a proposed or existing attribute acceptance-sampling plan. A possibility of immediate interest is the use of a plan from the MIL-STD-105B Tables.

Suppose, for example, that a 105B plan with an Acceptable Quality Level (AQL) of 2.5% and Sample Size Letter J has been proposed for use. Reference to Table IV-A of the 105B Standard shows that for single sampling a sample size of 75 items and an acceptance number of 4 is specified. Suppose life testing time is to be 80 hours with simply a count made of the test items failing by the end of that period. From inspection experience with the product to which the plan is to be applied, it seems most appropriate to assume a Weibull distribution with a value for β of 1 2/3. The lot size will be relatively large compared to the sample size of 75 so binomial probabilities for sample items can be assumed. Actually, Table III of MIL-STD-105B specifies that the lot size should be from 1300 to 3200, 501 to 800, and 181 to 300 for Inspection Levels I, II, and III respectively.

The first step is to determine the probability of acceptance, P(A), for various values of p'. These probabilities can easily be obtained from any one of the readily available tables of the cumulative binomial terms or tables of the incomplete beta distribution. They may also be read from the operating characteristic curves published as a part of the MIL-STD-105B Tables (Table VI). A few of these values for this plan are shown in the first and second columns of the tabulation below. Next, the first of the conversion tables, Table 1, is used to obtain values for the ratio $(t/\mu) \times 100$ for each of the p' values. These table values are listed in the third column. Finally, using the value for t of 80 hours,

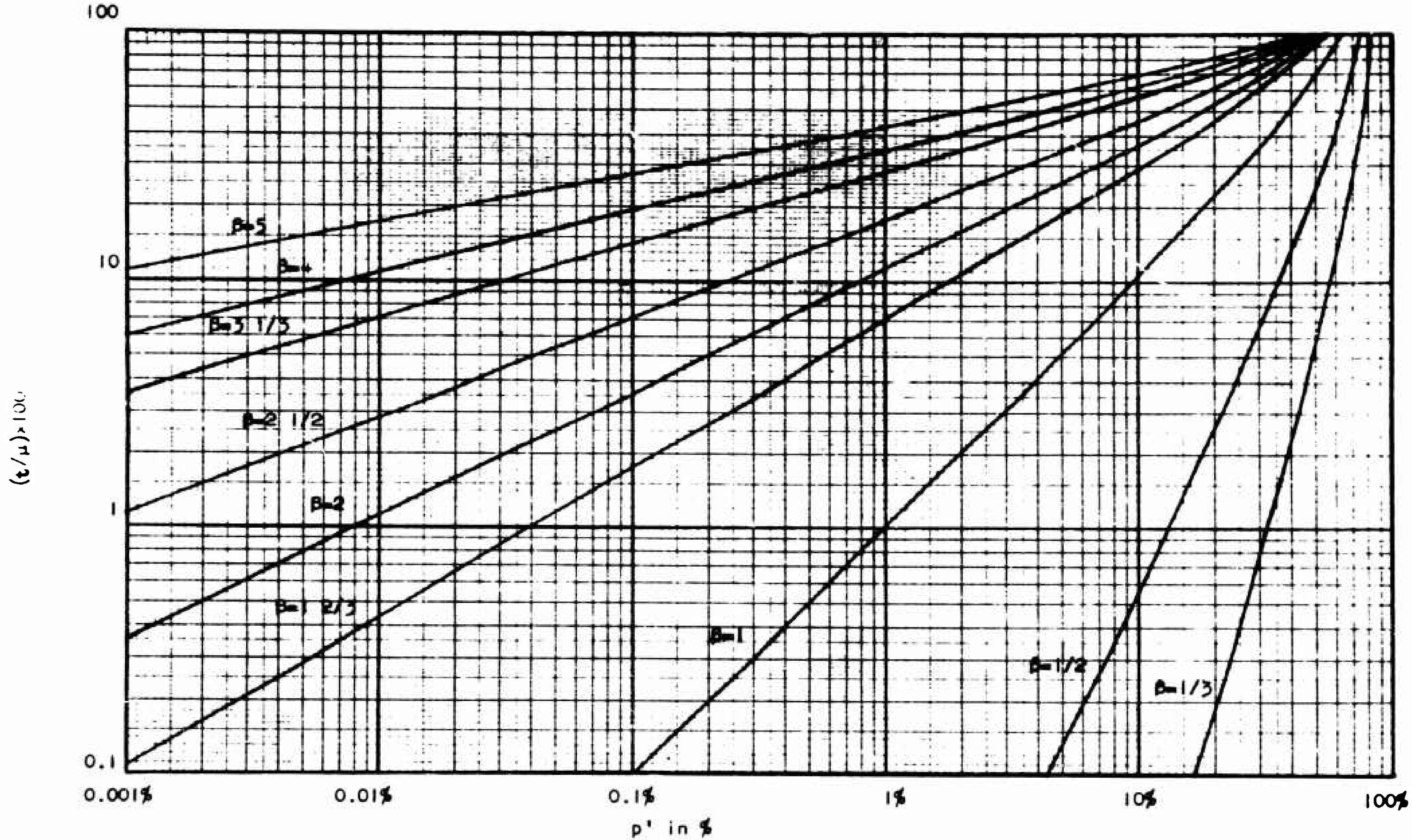

Figure 2. The Relationship Between $(t/\mu)\times 100$ and p' for Various Values of β

each of the $(t/\mu) \times 100$ ratios are converted to values for μ. For example, the ratio for a p' of 5% is 18.84. Thus $(80/\mu) \times 100 = 18.84$ or $\mu = 425$ hours. These computations have been made with results as shown in the last column of the tabulation. One may now note that if a lot is submitted to this plan whose mean life is 215 hours, the probability of its acceptance is only .01 or one in a hundred; on the other hand, if the mean life for a lot is 745 hours the probability of acceptance is .98. These probability and mean life figures based upon $t = 80$ hours can be plotted, if desired, to give the operating characteristic curve. (Of course similar OC curves for other known values of t may be plotted.) This curve is the one shown for $\beta = 1\ 2/3$ in Figure 3.

Results of Computations - Example 1

p' (in %)	P(A)	$(t/\mu) \times 100$	μ
2	.98	10.77	745
3	.92	13.78	580
4	.82	16.42	490
5	.68	18.84	425
6 1/2	.46	22.15	360
8	.27	25.20	315
10	.12	29.01	275
12	.04	32.58	245
15	.01	37.63	215

To indicate the importance of considering the shape of the life density for a product, operating characteristic curves for this plan have been computed and plotted in Figure 3 for other selected values for β. Included is a curve for the case in which β equals 1. This represents the exponential distribution widely used as a model in reliability and life-testing sampling inspection. From these curves it may be noted that if the underlying distribution is actually of the Weibull form and the exponential is assumed, the actual operating characteristics of the plan may differ very much from those contemplated. A discussion of the sensitivity of statistical procedures in current use to departures from the assumed exponentiality will be found in a paper by Zelen and Dannemiller[17].

It may be noted in connection with this example that the MIL-STD-105B plans include matching double and multiple sampling plans. These offer alternative possibilities for reliability and life testing applications. If incoming lots are either quite good or quite bad (as is commonly the case), substantial reductions in the number of items that must be tested may be made. If items are expensive and if testing is destructive (as it most likely will be in life testing), a

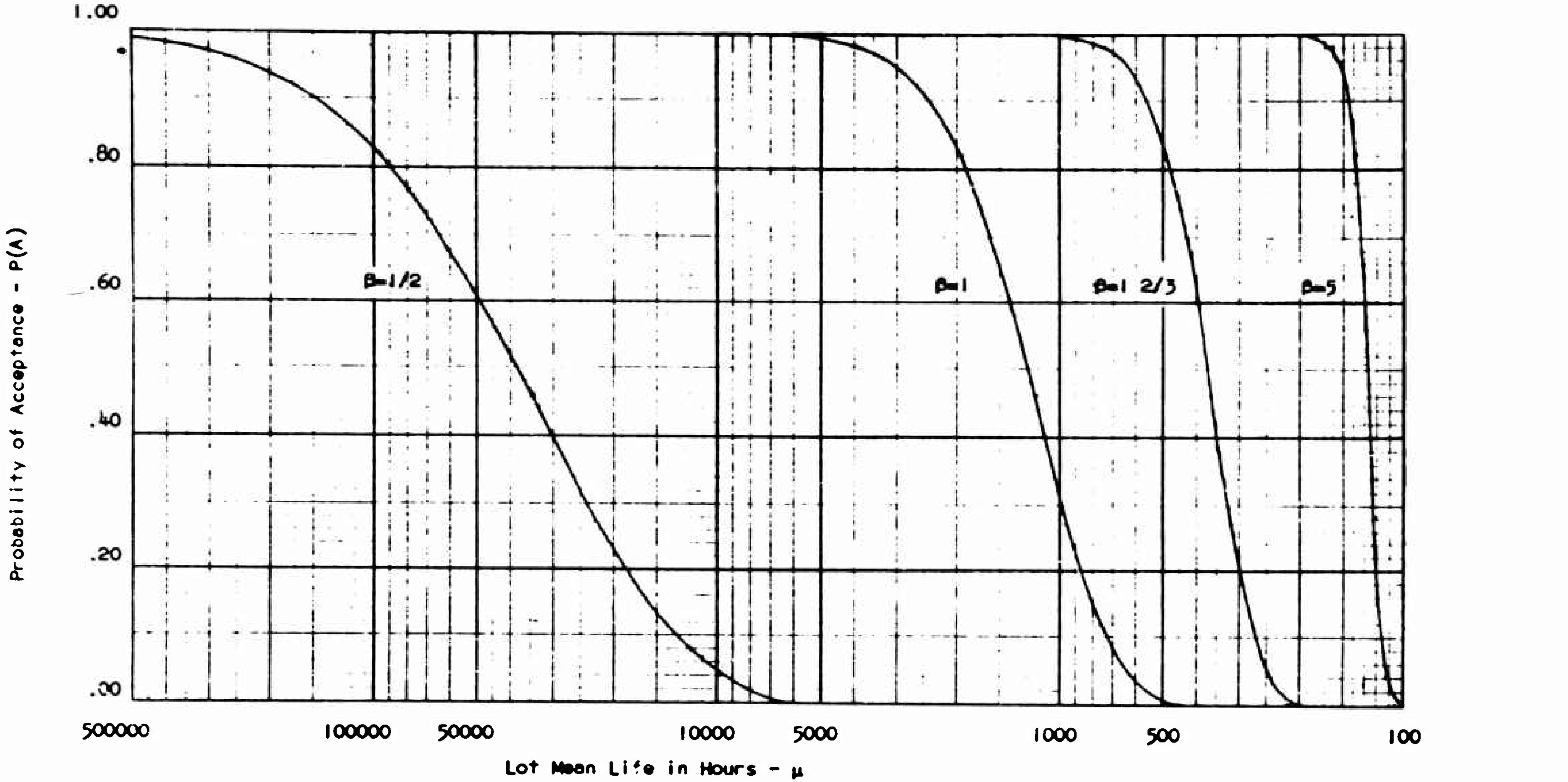

Figure 3. Operating Characteristic Curves for Various Values of β

$n = 75$ $c = 4$ $t = 80$ hrs.

reduction in average sample size may be of importance. If the test period, t, is relatively long, however, the elapsed time required for testing a second sample (or subsequent ones in multiple sampling) when such samples are required to reach a decision may raise ifficulties.

Example (2)

For a second example, consider the case of a manufacturer who knows that his current production of a certain component has a mean life of approximately 52,000 cycles. Furthermore, he has learned from past experience with life testing of these components that he can assume a β value of 1/2. A life test period of 1000 cycles seems justifiable and facilities are available for testing a sample of 150 items from each lot. This manufacturer would like to know what acceptance criteria to apply so that virtually all lots will be passed as long as the expected mean life of 52,000 cycles is maintained. He would also like to know what consumer protection will be afforded. A final question is whether for this application a change to a proposed test period of 300 cycles and a sample size of 500 items would yield comparable or better quality assurance.

The first step toward answers to these questions is to compute the $(t/\mu) \times 100$ ratio at the mean life considered acceptable. This ratio is $(1000/52{,}000) \times 100$ or 1.93. Entering Table 2 with this value gives (with rough interpolation) a value for p' of 18% for a β value of 1/2. Assuming a probability of acceptance of .95 is desired for lots at the acceptable quality level of 52,000 cycles for the lot mean, entering a table of the cumulative binomial distribution indicates that an acceptance number, c, of 35 items gives this probability for a sample size of 150 items. This, then, is the desired acceptance criteria.

A simple measure of consumer's protection is to find the lot mean value at which lots will likely be rejected. Suppose a probability of rejection of .90 (of acceptance of .10) seems to be a meaningful figure. Reference again to a binomial table indicates that for $n = 150$ and $c = 35$, the probability of rejection is .90 at a p' of approximately 28.4%. Entering Table 2 with this value gives a $(t/\mu) \times 100$ ratio of approximately 5.7. Substituting a value of 1000 cycles for t in this ratio and solving for μ gives a lot mean of 17,500 cycles. This figure for consumer's protection can be interpreted as follows--since this quality ($\mu = 17{,}500$) corresponds to a $P(A) = .10$, under this sampling plan ($n = 150$, $c = 35$) on the average 90% of the lots passed to the consumer will have a mean life of no less than 17,500 cycles. This may or may not represent adequate consumer protection. If it does not, a plan with a larger sample size must be designed and used.

An answer to the third question may be found by making similar computations for an n of 500 items and a value for t of 300 cycles. In this case $(t/\mu) \times 100$ equals $(300/52{,}000) \times 100$ or .58. From Table 2 it is found that p' is approximately 10% at this truncation ratio. Scanning a binomial table indicates an acceptance number of 62 will give a probability of acceptance of .95 or more when the sample size is 500 items. With this sample size and acceptance number, the probability of rejection is .90 at a p' value of approximately 14%. With this value for p', a $(t/\mu) \times 100$ value of approximately 1.15 is found from Table 1. Substituting 300 cycles for t in this ratio gives a lot mean value of 26,100 cycles as compared to 17,500 cycles for the first plan. Thus this combination of sample size and length of test period gives better discrimination between good and bad lots and the consumer is therefore better protected.

Example (3)

In this example, reference will be made to a case for which the component life can best be characterized by a mixture of two Weibull distributions. Kao[13] gives an example of this for the life of electron tubes. From electron tube life experience, the wearout failures, i.e., drift of electrical properties beyond some set limits, invariably occur near the latter part of life. Hence the failures of electron tubes are classified both as of the wearout type and as of the non-wearout or catastrophic type, each type being represented by a sub-population of the whole. In electronic terms, these failure types are referred to as electrical rejects and inoperative rejects respectively. The catastrophic (or inoperative rejects) sub-population is assumed to start at time zero, i.e., the location parameter $\gamma_1 = 0$, when the components are first exposed to risks. The wearout (or electrical rejects) sub-population is assumed not to start until some delayed period has elapsed, i.e., $\gamma_2 > 0$, since the limits set on the component drift depend on many factors such as environmental stress, maintenance policy, legal regulations, and like factors. Since, in general, failures due to wearout and to non-wearout reasons are identifiable, it is possible to treat the two sub-populations separately.

Suppose that for some application of electron tubes, the manufacturer's past experience indicates that the Weibull shape parameter β_1, associated with the catastrophic sub-population has a value of 1/2, and the shape parameter associated with the wearout sub-population, β_2, has a value of 3 1/3. Suppose further that electrical drift or wearout failure has never been experienced prior to 1000 hours of life. Under these conditions the location parameter values will be: $\gamma_1 = 0$ and $\gamma_2 = 1000$. Suppose further that the manufacturer knows that approximately 2 1/2% of the total tube failures are of the inoperative type and that the

mean tube life for his current production is,

$$\mu = (.025)(25,000) + (.975)(11,000) = 11,325 \text{ hrs.}$$

where $\mu_1 = 25,000$ is the mean life of the catastrophic sub-population and $\mu_2 = 11,000$ is that of the wearout sub-population. (See the appendix of the paper by Kao[13] for the derivation of this formula.) A life test period of 500 hours for inoperatives and of 5000 hours for electrical drifts are recommended and acceptance numbers $c_1 = 2$ and $c_2 = 2$ for each failure type are considered satisfactory. What are the necessary sample sizes so that the producer's risk is no more than 5%? Also, what would be the consumer's protection under this sampling plan?

To answer these questions, the two sub-populations are treated separately and are denoted by subscripts 1 and 2 as done before for the inoperatives and electrical drifts respectively. For inoperatives, $(t_1/\mu_1) \times 100 = (500 \times 100)/25,000 = 2.0$. Entering Table 2 with this value gives a value for p'_1 of 18.13% for a β value of 1/2. From a binomial table with $P(A) \geq .95$ and $p'_1 = 18.13\%$ a value for $n_1 = 5$ is obtained. The same binomial table for $n_1 = 5$, $c_1 = 2$ and $P(A) \leq .10$, gives $p'_1 = 75\%$. Entering Table 2 with this value gives $(t_1/\mu_1) \times 100 = 96.5$ and $\mu_1 = (500 \times 100)/96.5 = 518.24$ hours, a value which will be commented on later. For electrical drifts, γ_2 must be subtracted from t_2 and μ_2 giving new values for t_2 and μ_2 equal to 4,000 and 10,000 respectively. Hence, $(t_2/\mu_2) \times 100 = (4,000 \times 100)/10,000 = 40.0$. Entering Table 2 with this value gives for p'_2 a figure of 3.25% for a β value of 3 1/3. From a binomial table with $P(A) \geq .95$ and $p'_2 = 3.25\%$, it is found that $n_2 = 25$. The same binomial table for $n_2 = 25$, $c_2 = 2$, $P(A) \leq .10$ gives $p'_2 = 20\%$. Entering Table 1 with this value gives $(t_2/\mu_2) \times 100 = 71.04$. Thus $\mu_2 = (4,000 \times 100)/71.04 = 5,631$ hours, which upon readding γ_2 gives 6,631 hours. Combining this value of the corrected μ_2 with μ_1 obtained for inoperatives gives the consumer's protection expressed in terms of a mean value equal to,

$$\mu = (.025)(518.24) + (.975)(6,631) = 6,478 \text{ hours.}$$

This means under the sampling plan of running a life test for inoperatives of $n = 5$ and $c = 2$ for 500 hours and another life test for electrical drifts of $n = 25$ and $c = 2$ for 5000 hours, 90% of the lots passed to consumers will have a mean life of at least 6,478 hours. To illustrate the danger of extrapolation in a mixed distribution case, assume that the second test of 5000 hours duration was not run at all. Then the producer could only base his conclusion upon the

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500-hour test and claim as consumers' protection, with 90% confidence, a mean life of at least 518.24 hours, a result which is altogether too modest.

SECTION 3
THE TABLES OF SAMPLING PLANS

3.1 Construction and Use of the Basic Tables.

A basic set of tables of sampling inspection plans has been prepared, one table for each of the nine values for β for which the relationship between p' and $(t/\mu) \times 100$ has been established. These are presented near the end of this report as Tables 3a through 3i.

Each table lists values for the acceptance number, c, and for the minimum sample size, n, for a variety of objective t/μ ratios. The plans are designed so that if 100 times the ratio between the test time, t, and the mean life value for the lot, μ, or $(t/\mu) \times 100$ is equal to or greater than the selected column value in the table, the probability of acceptance, P(A), will be .10 or less. Stated otherwise, the plans assure with 90% confidence or more the acceptance of lots for which the $(t/\mu) \times 100$ ratio is equal to or less than the selected column or objective value. The ratios in the column headings (for which the plans have been designed) may thus be considered in the same way as lot tolerance per cent defective (LTPD) values are considered in describing operating characteristics of the widely used attribute and variables acceptance plans.

It has been assumed that in acceptance inspection for reliability the consumer's risk will be of primary concern. For this reason, these plans have been cataloged by P(A) = .10 ratios which measure consumer protection. However, in addition for each plan the $(t/\mu) \times 100$ ratio is given for which the probability of acceptance is .95 or more. Each such P(A) = .95 ratio value is shown in parentheses under the corresponding sample size number. These ratio values may be considered similar to acceptable quality level (AQL) values in indicating the producer's risk. If the mean life for the items in the lot is such that the $(t/\mu) \times 100$ ratio is equal to or less than the tabulated value, there is assurance with confidence of 95% or more that the lot will be accepted.

The two ratio values, one in the column heading and the other in the body of the table in parentheses below the sample size number, broadly describe the operating characteristics of each plan and so form a basis for making an appropriate choice for any acceptance inspection application. These values may also be used to determine approximately the operating characteristics of any acceptance plan that has been specified or that is in use and for which n and c match closely one of the table plans. It is easy to convert these ratios to hours, cycles or some other measure of lifelength to fit the product and test

specifications involved. Thsi will be illustrated by two examples which follow later.

In the preparation of these plans, binomial tables prepared by Grubbs[18] were employed for values for c up to 9 and for n up to 150. For higher values of c and for values for n up to 60 or so, the Pearson tables of the incomplete beta-function were used[19]. Higher values of n were determined by the Poisson approximation, using a table of np' values prepared by Cameron[20]. The Poisson match was checked and was found close, even for the smaller sample sizes and the larger values for p'. The slight differences that may exist in some cases is on the conservative side; the value for n is slightly larger than that theoretically required. As the tables were prepared as a part of an exploratory study, plans showing extremely large sample sizes have been included to indicate the order of magnitude involved and not with the expectation that samples of this size would ordinarily be used in acceptance-inspection practice.

3.2 Examples of Application of the Tables.

Example (4)

Suppose an acceptance inspection plan is required which will assure with 90% confidence a mean life for items of 4000 hours or more for each lot accepted. Also, suppose it will be desirable to assure the producer that if the mean life for items in a lot is 25,000 hours or more, there will be a high probability (.95) that the lot will be accepted. A test period of 400 hours for the inspection of sample items has been specified. Through past experience it has been determined that the distribution of item life is of the Weibull form with the shape parameter, β, equal to approximately 1/2. Also, a value for γ, the location parameter, of 0 can be assumed.

For these sampling plan specifications, 100 times the ratio of test time, t, to the mean life, $\mu_{.10}$, for which a probability of acceptance of .10 or less is desired is $(400/4{,}000) \times 100$ or 10. At the .95 probability value the ratio is $(400/25{,}000) \times 100$ or 1.6. A plan approximately meeting these requirements may be found in Table 3b which gives plans for distributions for which $\beta = 1/2$. The column for which $(t/\mu) \times 100 = 10$ is entered and then scanned for the ratio value 1.6 (or one close to this value) among the values in the column listed in parentheses. This value is found well down in the column. The corresponding sampling inspection plan specifies a sample size, n, of 43 and an acceptance number, c, of 11.

Example (5)

Suppose that a sampling inspection plan specifies that a random sample of 3000 items is to be drawn from the lot and tested for a period of 2,480 hours If no more than 7 items fail before the end of the test period, the lot is to be accepted; if more than 7 items do not live through the test period, the lot is to be rejected. Life measurements for past inspection and research for the product to which the plan is to be applied indicate the distribution is of the Weibull form with β equal to approximately 1 2/3. Also, a value for γ, the location parameter, of zero is indicated. The prospective user of this plan would like to know what quality protection will be given. Inspection of Table 3d which lists plans for β = 1 2/3 discloses a plan matching reasonably well the one specified, the plan for which c, the acceptance number, is 7 and n, the sample size, is 3,019. For this table plan the $(t/\mu) \times 100$ ratio at P(A) = .10 is 4. Substitution of the specified test period length of 480 hours for t gives $(480/\mu) \times 100 = 4$. Solving for μ gives 12,000 hours as the mean value for item life for the lot for which the probability of acceptance is .10 or less. A similar substitution for t using the ratio at which P(A) = .95 gives $(480/\mu) \times 100 = 2.1$. Solving for μ again gives 23,000 hours as a lot mean value for which the probability of acceptance is .95 or better. The values for the lot mean at these two probability values broadly, but very practically, describe the operating characteristics of the specified plan.

3.3 Some Points of Practice.

In the use of these tables of plans, several points of practice should be noted. First, in using the p' values associated with values for $(t/\mu) \times 100$ for the Weibull distribution to find values for c and n, the binomial probability distribution has been used. Hence the size of the lot should be relatively large compared to the size of the sample for the stated probability values to precisely apply. Second, if a plan is not available for which a $(t/\mu) \times 100$ ratio in the column headings matches closely the desired ratio, to be conservative, a plan should be chosen from the column with the next smaller ratio heading. This will assure with confidence greater than 90% the specified mean life for acceptance. If the acceptable quality level (the ratio or mean life for which P(A) = .95) must also be guaranteed and a matching ratio value is not found in the selected column of plans, a plan with the next greater value should be selected. Lots equal to or better than the specified "acceptable quality" will have an assurance of greater than 95% of being accepted. With proper care, some rough interpolation may be employed between listed sample sizes (either down or

across the table or both) to find a new plan having more nearly the desired characteristics. Finally, if a plan is found for which the desired or given ratios closely match but for practical reasons it seems desirable to round off the sample size to the nearest number ending in zero or five, such rounding off should be done to the next larger size. This will assure retention of the probability values of .10 or less for the ratios given in the column headings.

SECTION 4

PLAN DESIGN IN TERMS OF QUALITY LEVEL RATIOS

4.1 The Relationship Between Acceptable and Unacceptable Lot Quality.

Sampling plans are most conveniently cataloged, selected, or designed in terms of a producer's risk and a consumer's risk. Some lot quality figure will be specified as satisfactory and for lots of this quality or better the probability of acceptance should be high, conventionally .95 or more (the producer's risk or rejection small, .05 or less). For the plans for life testing, described in Section 3, this specification will be a lot mean life, $\mu_{.95}$, at which $P(A) \geq .95$. Likewise, an unsatisfactory quality level will be specified for which the probability of acceptance will be low, conventionally .10. This specification will be a lot mean life, $\mu_{.10}$, at which $P(A) \leq .10$. (If other values of producer's and consumer's risks must be determined for these plans, reference may be made to a method in a paper by Kao[21].)

In plan selection or design, one objective is to find a combination of sample size and acceptance number which simultaneously yields the desired values for both the consumer risk and the producer risk. If working from tables of plans, the values for lot quality at the two risk figures may be found listed in the tables. In the design of a plan, one may cut and try until a suitable plan is found in a manner suggested in Example 2. Also, factors are available which, in conjunction with the conversion tables supplied here as Table 1 and Table 2, enable a direct determination to be made[20]

A simple alternative solution for the form of acceptance inspection discussed here is to make use of one of its properties, namely that for a given acceptance number, c, (and for a given value for β) the ratio between the lot means at the two risk values is approximately constant for all values of sample size, n. These ratios (or multipliers) have been determined as part of the study underlying this report for values for c ranging from 0 to 15 for each of the various values for β. They are presented in Table 4. The table values are in the form of multipliers for finding $\mu_{.95}$, given $\mu_{.10}$, or by using the reciprocal of the multiplier, for finding $\mu_{.10}$, given $\mu_{.95}$. That is, $\mu_{.95}$ (for which $P(A) = .95$) is equal (approximately) to $\mu_{.10}$ (for which $P(A) = .10$) times the appropriate table multiplier. These multipliers may be used both to assist in evaluating the operating characteristics of some given plan and to assist in the design of a plan to meet some acceptance-inspection requirement.

4.2 An Example of Use of the Table of Multipliers.

Example (6)

For a certain purchased component the lot mean life should be at least 4,000 hours; this value is accordingly chosen for $\mu_{.10}$. Also, the producer has been informed that lots whose mean life is 10,000 hours or more are reasonably sure of acceptance through the sampling procedure. Accordingly, this value is to be used for $\mu_{.95}$. A value for β of 1 can be assumed. A testing period, t, of 200 hours has been specified. Values for sample size, n, and acceptance number, c, must be found to meet these requirements.

The ratio between the two lot means, $\mu_{.95}/\mu_{.10}$, is 10,000/4,000 or 2.5. Examination of the table of mean life multipliers, Table 4, under the column for $\beta = 1$ indicates that an acceptance number, c, of 10 items will give this ratio. The $(t/\mu) \times 100$ ratio at $\mu_{.10}$ is $(200/4{,}000) \times 100$ or 5. Entering Table 2, the table of p', with this truncation ratio value of 5, gives a p' of 4.88%. Reference to a table of the cumulative binomial distribution or use of the Poisson approximation for c = 10 and p' = .0488 at P(A) = .10 shows that a sample size, n, of 315 items meets the requirements. A check for this solution can be made, if desired. For n = 315, c = 10, and P(A) = .95 the Poisson approximation indicates a p' of 1.96%. Entering Table 1, the table for per cent truncation, with this value for p', a value for $(t/\mu) \times 100$ of approximately 2.0 is found. Solving for $\mu_{.95}$ yields $(200/\mu) \times 100 = 2.0$ or $\mu_{.95} = 10{,}000$ which is the desired value.

SECTION 5

AN ADAPTATION OF THE MIL-STD-105B PLANS

5.1 Use of the 105B Plans for Life and Reliability Testing.

To permit use to be made of the familiar Military Standard 105B plans for life and reliability testing application, the conversion factors described in Section 2 of this report were employed to find $(t/\mu) \times 100$ ratios for all of the plans in the 105B collection. As for the basic plans described in Section 3, separate tables have been prepared for each of a number of selected values for β, the Weibull shape parameter. The special case for $\beta = 1$, which is the exponential case, has been included.

The acceptance procedure will be the same as that employed for the basic plans and as outlined in Section 1.2: (1) a random sample of n items is selected from the lot, (2) the sample items are tested for life over some preassigned test period length, t, (3) the number of test items failing prior to time t is observed, (4) if the number of test items failing is no more than some specified acceptance number, c, the lot is accepted - if more, the lot is rejected. Lot quality is evaluated in terms of mean item life, μ. Both t and μ are measured from some reference time or the value for the Weibull location parameter, γ.

The sample sizes and acceptance numbers used will be those specified by the MIL-STD-105B tables. The procedure as outlined here is for single sampling; through simple and appropriate modification, the 105B double-sampling and multiple-sampling plans may be likewise employed. Example 7 in a following subsection of this report discusses this possibility. It may be noted that the acceptance procedure (for any form of sampling) is the same as that specified for the MIL-STD-105B plans with the single exception of the use of a test truncation time, t, (and the application, of course, to life-testing data).

As discussed in Section 2, the probability of acceptance for a lot under the acceptance procedure outlined depends solely upon the probability, p', that an item will fail before the end of the test period, t. If the test truncation time, t, is preassigned and if the value for β, the shape parameter, is known the probability, p', of failure is a function only of mean item life, μ. This fact makes it possible to use this attribute acceptance procedure to evaluate lots in terms of mean item life; the operating characteristics for any specified plan (in terms of c and n) depend only on t and μ.

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It should be noted again that it is necessary to assume a value for β, the Weibull shape parameter. For many applications this value may be known. Its magnitude may have been determined for the product in question from past life-length research data, from the results of past inspection data, or from some other source. If the value for β is not known, procedures are available, as outlined in Appendix B, for estimating this parameter (and the γ location parameter also if this, too, is necessary).

5.2 <u>The 105B Tables of Ratios.</u>

Tables of ratios for adapting the 105B plans to reliability and life-testing application have been prepared for each of seven typical values for β. These seven values range from 1/2 to 3-1/3, covering the span commonly encountered with industrial products. A table for $\beta = 1$, which is the exponential case, is also included. These tables will be found at the end of this report as Tables 5a through 5g.

In order to allow for any desired test-time truncation value, t, and to make the plans available for general use, the tables have been prepared in terms of the dimensionless ratio, t/μ. Actually, to give more conveniently usable figures and to work in terms adopted for the 105B plans, the ratio is given in terms of percent; $(t/\mu) \times 100$ is used. Each of the 105B plans is cataloged and described in terms of the $(t/\mu) \times 100$ ratio. These ratio values are used in the same way as the percent defective values are used in the selection and application of the 105B plans for ordinary attribute inspection. When applying the plans to a specific life-testing application, use of the ratio to convert from test time to lot mean (or vice versa) will be found quite easy. Examples of application are given in Section 5.3.

Each table lists for each 105B Acceptable Quality Level (AQL) value the corresponding $(t/\mu) \times 100$ ratio. These matched ratios will be found in the column headings under each of the respective 105B Acceptable Quality Level values (which are in terms of 100 p' %, the acceptable percent defective) Each ratio value gives for all 105B plans of the corresponding AQL, a measure of lot quality for which the producer's risk or probability of rejection will be low. This risk of rejection will be the same as that encountered in the normal use of the 105B plans for attribute inspection. It will be recalled that this risk is not a constant value of, say .05, as in most previous tables of acceptance inspection plans, but ranges from as low as 0.01 to as high as 0.20. The risk varies with the size of the sample, which in turn varies with the size of

the lot. For large lot sizes (and thus large sample sizes) the risk is relatively small; for small lot sizes it is relatively large. The specific risk value for any plan of interest may be obtained from the corresponding operating characteristic curve which will be found included with the 105B tables.

The interpretation of these matched Acceptable Quality Level ratios for life-testing and reliability use may be demonstrated by means of a specific case. Assume, for example, that β = 1-2/3 and that a 105B plan with an AQL of 4.0% is to be used. From the table of ratios for β = 1-2/3, which is Table 5e, it will be found that the corresponding t/μ ratio value at the AQL is 16.42. Thus lots for which $(t/\mu) \times 100 = 16.42$ are "acceptable" and the probability of acceptance will be high (the probability of rejection low). If the test period, t, is , say, 1000 hours, $(100/\mu) \times 100 = 16.42$ or μ = 6,090 hours; the mean life for the items in the lot must be 6,090 hours for it to meet the 105B acceptable quality level standards.

In the body of each table of conversion ratios will be found for each 105B plan ratios for which the probability of acceptance is .10. These correspond to lot tolerance per cent defective (LTPD) figures which furnish a useful measure of consumer's protection. They represent unsatisfactory lot quality values and ones for which the probability of acceptance is low. Unlike the risks associated with the AQL which vary with sample size, the risk at the LTPD quality used for the tables presented here is at .10 for all plans.

For the example cited above in which the AQL is 4.0%, suppose the Sample Size Code Letter to be used is L. Reference to the table of conversion ratios for β = 1-2/3 shows the LTPD ratio to be 31. With the test period, t, designated as 1000 hours, the value for μ, the lot mean life, may be readily determined. Substitution gives $(1000/\mu) \times 100 = 31$ or μ = 3,220 hours. Thus lots whose mean life is 3220 hours or less have a probability of at most .10 of acceptance.

With the use of these complete tables of conversion ratios (tables 5a through 5g), suitable 105B plans may be selected in terms of either an AQL or the LTPD (or both, if desired). If, instead, some 105B plan has been specified, its operating characteristics can be evaluated. Examples of such use will be outlined in the material that follows.

As a supplement to these tables, Table 6 has been prepared. This table gives the $(t/\mu) \times 100$ ratio at the Acceptable Quality Level for an additional number of values for β, the Weibull shape parameter. Ratios at the AQL for the β values used in Table 5 are also included for convenience. As the Acceptable Quality Level supplies the operating characteristic of most interest in the application of 105B plans, the ratio values in this table may be all that are

necessary for many applications. Ratios are given for seven additional values for β for which conversion factors were available. Also included is a table, Table 7, of the sample sizes and acceptance criteria for the Military Standard 105B plans for single sampling.

5.3 Examples of Application.

Example (7)

For a simple example of application, consider a receiving inspection case for which incoming lots of a product are to be tested for lifelength by sampling. From past experience with the product it has been determined that the life distribution can be expected to follow the Weibull form with a value for β, the shape parameter, of approximately 1-1/3. The value for γ, the location parameter, is expected to be 0. The MIL-STD-105B plans are to be employed. A test period for the sample items of 200 hours and an Acceptable Quality Level in terms of percent defective (as used in the Standard) of 1.5% have been more or less arbitrarily selected for use. The size of incoming lots is 5,000 items. Inspection Level II, the one for normal use, will seemingly be appropriate. Single sampling is to be employed. The acceptance procedure for the above conditions and the resulting operating characteristics must be determined.

Reference to Table III of MIL-STD-105B shows that for a lot size of 5,000 items and for Inspection Level II, Sample Size Code Letter M is designated for ordinary inspection. Reference is next to Table IV-A of the Standard, the master table for normal single-sampling inspection or to Table 7 of this report. Here it will be found that for Sample Size Code Letter M and for an Acceptable Quality Level of 1.5%, the sample size is 225 items, the acceptance number is 8 items, and the rejection number is 9. The acceptance-rejection procedure will thus be the following: (a) draw at random from the submitted lot a sample of 225 items and place them on life test for 200 hours, (b) determine the number of items that have failed by the end of this test period, (c) if the number failing is 8 or less, accept the lot. If the number is 9 or more, reject it.

The operating characteristics of this plan can be determined from information included in the tables of $(t/\mu) \times 100$ ratios included as part of this report. For this example reference will be to Table 5d, the table of ratios for $\beta = 1\text{-}1/3$. Examination of the two lines of Acceptable Quality Level values across the top of this table shows that for an Acceptable Quality Level in terms of p' (%) of 1.5, the Acceptable Quality Level in terms of $(t/\mu) \times 100$ is 4.69. With this latter ratio, and with the value for t, the test period length of 200 hours, the

value for μ, the mean item life for the lot, can be determined. Thus:

$$(t/\mu) \times 100 = 4.69 \qquad \text{(AQL)}$$
$$(200/\mu) \times 100 = 4.69$$
$$\mu = 4{,}260 \text{ hours.}$$

One now knows that the operation of the plan is such that if the mean item life for the lot is 4,260 hours or more the probability that it will be accepted is high. (A rough value for this probability may be found from the operating characteristic curves in the Military Standard. For an AQL of 1.5 and for Code Letter M one may note the probability is approximately .97.) The Acceptable Quality Level is thus 4,260 hours.

The ability of the plan to protect the consumer may be measured by the lot mean life for which the probability of acceptance is low. The tables of $(t/\mu) \times 100$ ratios included in this report include ratios at the Lot Tolerance Percent Defective (LTPD) quality level, the level at which the probability of acceptance is .10. These ratios will be found in the body of the tables. Reference to the same table, Table 5d for $\beta = 1\text{-}1/3$, gives an LTPD ratio of 13 for Sample Size Code Letter M and an AQL of 1.5. Computations similar to those previously made give:

$$(t/\mu) \times 100 = 13 \qquad \text{(LTPD)}$$
$$(200/\mu) \times 100 = 13$$
$$\mu = 1{,}540 \text{ hours.}$$

One now knows that if the mean life for items in a submitted lot is 1,540 hours or less, the probability of it being accepted is at most .10; the probability of its rejection is at least .90.

The operating characteristics in hours as computed above apply also (with the same values) if a double-sampling or a multiple-sampling plan for the same Sample Size Code Letter and AQL value is employed instead of a single-sampling plan. For double-sampling in this application, the data for the plan will be found in Table IV-B of the Military Standard. The first sample size will be 150 items. These sample items would be tested for 200 hours. If 5 or fewer items failed within this time, the lot would be accepted; if 14 or more failed, it would be rejected. If from 6 to 13 failed, a second sample of 300 items would be selected and tested for 200 hours. If the total number failing (in the first and second samples combined) is 13 or less, the lot would be accepted, if it is 14 or more, the lot would be rejected.

Example (8)

For a second example, consider an acceptance-inspection by sampling

application for which the following achievements are desired: (a) If the mean item life for the lot is 20,000 hours or more the probability of acceptance will be high. Lots of this mean life or greater are considered "acceptable." (b) If the mean item life for the lot is 6,000 hours or less, the probability of acceptance will be low, that is .10. A test period of 500 hours will be employed. It is expected that the item life distribution will be of the Weibull form with a value for β, the shape parameter of 3/4. The value for the location parameter, γ, is zero.

The $(t/\mu) \times 100$ ratio at the AQL will be

$$(500/20{,}000) \times 100 \text{ or } 2.5 .$$

The $(t/\mu) \times 100$ ratio at the LTPD will be

$$(500/6{,}000) \times 100 \text{ or } 8.3 .$$

With these values, Table 5b giving the $(t/\mu) \times 100$ ratios for $\beta = 3/4$ may be scanned to determine the appropriate MIL-STD-105B plan. One may note in this table that an AQL of 6.5 (in percent defective) corresponds to a $(t/\mu) \times 100$ ratio of 2.29. This is the closest value available for the desired ratio value of 2.5. Next, the column under the 6.5 AQL value heading may be scanned to find a close approximation to the desired value of 8.3 for the LTPD. A value of 8.2, which is reasonably close, is found corresponding to Sample Size Letter K.

Thus any MIL-STD-105B plan with Sample Size Code Letter K and with an Acceptable Quality Level of 6.5 will give approximately the desired operating characteristics for the specified test period of 500 hours. For single sampling, for example, the sample size will be 110 items and the acceptance number 12 as indicated in the MIL-STD-105B tables.

Example (9).

The procedure to be followed for cases in which the Weibull location parameter, γ, is not zero but is of some other known value may be illustrated by outlining a third example. The method to be followed in this case is to simply subtract the value for γ from the value for t, the test time, to get t_o, and from μ to get μ_o. These transformed values, t_o and μ_o are then used for all $(t/\mu) \times 100$ computations. The solution obtained in terms of t_o and μ_o can then be converted back to original values by simply adding the value for γ to each.

Consider, for example, an application for which a single-sampling plan with n equal to 35 and c equal to 1 has been specified. This corresponds to a plan with Sample Size Letter H and an AQL (in terms of p') of 1.5% in the 105B collection. Item life is measured in terms of cycles of operation. Protection

against lots for which the average item life is less than 5,000 cycles is required. From experience with this product it has been determined that the Weibull distribution applies and that a value for γ of 2,000 cycles and a value for β or 2 can be expected. The problem is to determine a test time, t, in cycles that will enable the plan to meet the above requirement for consumer's protection. A related problem is to find whether the plan so determined will give adequate producer protection. It has been determined that a mean item life of 16,000 can reasonably be expected from a competent supplier.

The first step toward a solution is to convert the required lot mean life, μ to a transformed value, μ_o. This new value, μ_o, is $\mu - \gamma$ or 5,000 - 2,000 which is 3,000 cycles. Next, from Table 5f which gives conversion ratios for use when $\beta = 2$, one finds that for Sample Size Letter H and an AQL in p' (%) of 1.5, the ratio at the LTPD Quality Level is 38 and at the AQL it is 13.87. Since the plan is to be determined in terms of consumer's needs, the next step is to use the LTPD ratio to determine t_o. Thus $(t_o/\mu_o) \times 100 = 38$ or $(t_o/3{,}000) \times 100 = 38$. From this it is determined that t_o must equal 1140. By adding the value for γ (which is 2,000) to this latter figure, the required test time in absolute terms, t, of 3,140 cycles is obtained.

The related question of the reasonableness of this plan for the producer may be answered by substitution of the test time just determined in the conversion ratio for the AQL. The relationship is that $(t_o/\mu_o) \times 100 = 13.87$ or that $(1140/\mu_o) \times 100 = 13.87$. From this a value for μ_o of 8,240 cycles is obtained. This is then converted to original terms by the addition of the value for γ of 2,000 cycles. This gives an Acceptable Quality Level of 10,240 cycles. This is well below the level considered reasonable so no hardship will be imposed on the supplier.

TABLE 1

Table of Values for Per cent Truncation, $(t/\mu) \times 100$

p'	Shape Parameter - β								
(in %)	1/3	1/2	1	1 2/3	2	2 1/2	3 1/3	4	5
.010			.010	.45	1.13	2.83	7.03	11.03	17.26
.012			.012	.49	1.24	3.04	7.42	11.55	17.91
.015			.015	.57	1.38	3.32	7.94	12.21	18.72
.020			.020	.67	1.59	3.73	8.66	13.12	19.83
.025			.025	.77	1.78	4.08	9.26	13.87	20.74
.030			.030	.86	1.95	4.40	9.77	14.52	21.50
.040			.040	1.02	2.26	4.93	10.65	15.60	22.77
.050			.050	1.18	2.53	5.39	11.40	16.49	23.82
.065			.065	1.37	2.88	5.98	12.32	17.62	25.10
.080			.080	1.56	3.19	6.50	13.13	18.56	26.16
.100			.10	1.78	3.57	7.11	14.03	19.62	27.36
.12			.12	1.98	3.92	7.65	14.82	20.53	28.37
.15			.15	2.26	4.37	8.36	15.84	21.71	29.67
.20			.20	2.69	5.07	9.39	17.27	23.33	31.43
.25			.25	3.08	5.64	10.27	18.47	24.68	32.87
.30			.30	3.44	6.18	11.05	19.51	25.83	34.09
.40			.40	4.07	7.14	12.39	21.27	27.76	36.12
.50		.001	.50	4.67	7.99	13.55	22.75	29.36	37.76
.65		.002	.65	5.46	9.12	15.06	24.62	31.35	39.81
.80		.003	.80	6.19	10.11	16.36	26.21	33.03	41.50
1.00		.005	1.01	7.08	11.31	17.90	28.03	34.93	43.40
1.2		.007	1.21	7.90	12.40	19.26	29.62	36.57	45.02
1.5		.011	1.51	9.07	13.87	21.08	31.68	38.68	47.09
2.0		.020	2.02	10.77	16.03	23.67	34.56	41.59	49.90
2.5		.032	2.53	12.33	17.95	25.90	36.98	44.01	52.21
3.0		.047	3.05	13.78	19.69	27.89	39.09	46.09	54.17
4.0	.001	.083	4.08	16.42	22.79	31.35	42.69	49.59	57.45
5.0	.002	.13	5.13	18.84	25.58	34.35	45.71	52.50	60.13
6.5	.005	.23	6.72	22.15	29.25	38.28	49.57	56.18	63.46
8.0	.010	.35	8.34	25.20	32.59	41.72	52.88	59.29	66.26
10.0	.020	.56	10.54	29.01	36.63	45.82	56.73	62.85	69.44
12	.034	.82	12.78	32.58	40.34	49.50	60.11	65.96	72.18
15	.070	1.32	16.25	37.63	45.48	54.49	64.60	70.05	75.73
20	.18	2.49	22.31	45.51	53.30	61.85	71.04	75.83	80.68
25	.40	4.14	28.77	52.99	60.53	68.47	76.67	80.80	84.89
30	.76	6.36	35.37	60.29	67.39	74.62	81.79	85.26	88.62
40	2.22	13.04	51.08	74.79	80.64	86.15	91.09	93.27	95.22
50	5.55	24.02	69.31	89.82	93.95	97.33	99.82	100.67	101.21
65	19.28	55.10	104.98	115.23	115.61	114.92	113.06	111.68	109.98
80	69.48	129.52	160.94	148.91	143.14	136.34	128.53	124.27	119.79

TABLE 2

Table of Probability Values at Truncation Point, p' (%)

(t/μ) x 100	Shape Parameter - β								
	1/3	1/2	1	1 2/3	2	2 1/2	3 1/3	4	5
.010	8.09	1.40	.010						
.012	8.57	1.54	.012						
.015	9.20	1.72	.015						
.020	10.08	1.98	.020						
.025	10.82	2.21	.025						
.030	11.45	2.42	.030						
.040	12.53	2.79	.040						
.050	13.43	3.11	.050						
.065	14.56	3.54	.065						
.080	15.52	3.92	.080						
.100	16.61	4.37	.10						
.12	17.56	4.78	.12	.001					
.15	18.78	5.33	.15	.002					
.20	20.46	6.13	.20	.003					
.25	21.86	6.83	.25	.004					
.30	23.06	7.45	.30	.005					
.40	25.06	8.56	.40	.009	.001				
.50	26.71	9.52	.50	.012	.002				
.65	28.76	10.78	.65	.019	.003				
.80	30.47	11.88	.80	.027	.005				
1.00	32.40	13.19	1.00	.038	.008				
1.2	34.03	14.35	1.19	.052	.011	.001			
1.5	36.12	15.90	1.49	.076	.018	.002			
2.0	38.94	18.13	1.98	.12	.031	.004			
2.5	41.22	20.04	2.47	.18	.049	.007			
3.0	43.14	21.73	2.96	.24	.071	.012	.001		
4.0	46.28	24.64	3.92	.39	.13	.024	.002		
5.0	48.80	27.11	4.88	.56	.20	.041	.003		
6.5	50.90	30.27	6.29	.89	.33	.080	.008	.001	
8.0	54.30	32.97	7.69	1.22	.50	.13	.015	.003	
10.0	56.98	36.06	9.52	1.77	.78	.23	.033	.007	.001
12	59.19	38.73	11.31	2.39	1.12	.37	.060	.014	.002
15	61.92	42.17	13.93	3.45	1.75	.64	.13	.034	.005
20	65.45	46.87	18.13	5.51	3.09	1.32	.33	.11	.021
25	68.17	50.69	22.12	7.89	4.79	2.29	.69	.26	.064
30	70.37	53.91	25.92	10.56	6.82	3.59	1.26	.55	.16
40	73.79	59.12	32.97	1[illegible].47	11.81	7.23	3.25	1.71	.67
50	76.36	63.21	39.35	22.98	17.83	11.28	6.72	4.13	2.02
65	79.28	68.02	47.80	33.26	28.24	22.32	15.37	11.35	7.29
80	81.49	71.77	55.07	43.54	39.51	34.59	28.35	24.15	19.25
100	83.75	75.69	63.21	56.35	54.41	52.36	50.41	49.08	47.93

TABLE 3a

Table of Sampling Plans for $\beta = 1/3$

c	n												
	$(t/\mu) \times 100$ Ratio for which P(A) = .10 (or Less)												
	100	50	25	10	5	2.5	1	0.5	0.25	0.1	0.05	0.025	0.010
0	1	2	2	3	4	5	6	8	10	13	16	21	28
1	3 (.05)	4 (.02)	4 (.01)	6	7	8	11	14	17	22	28	35	47
2	5 (.15)	5 (.16)	6 (.08)	8 (.03)	9 (.02)	12 (.01)	15	19	23	31	38	48	65
3	6 (.53)	7 (.27)	8 (.17)	10 (.07)	12 (.04)	15 (.02)	19 (.01)	24	29	39	48	60	81
4	7 (1.2)	8 (.66)	10 (.33)	12 (.14)	15 (.06)	18 (.03)	23 (.01)	28 (.01)	35	46	58	72	97
5	9 (1.3)	10 (.80)	11 (.54)	14 (.20)	17 (.10)	21 (.05)	27 (.02)	33 (.01)	41 (.01)	54	67	84	113
6	10 (2.2)	11 (1.2)	13 (.65)	16 (.28)	19 (.15)	24 (.07)	31 (.03)	37 (.02)	46 (.01)	61	76	95	128
7	11 (2.6)	13 (1.4)	15 (.76)	18 (.35)	22 (.18)	26 (.10)	34 (.04)	42 (.02)	52 (.01)	69	86	107	143
8	13 (2.9)	14 (1.7)	16 (1.1)	20 (.44)	24 (.23)	29 (.12)	38 (.05)	47 (.02)	58 (.01)	76	95	118	161
9	14 (3.8)	16 (2.1)	18 (1.2)	22 (.52)	27 (.24)	32 (.13)	41 (.06)	51 (.03)	63 (.01)	83 (.01)	103	129	176
10	15 (4.6)	17 (2.5)	20 (1.2)	24 (.58)	29 (.28)	35 (.14)	45 (.06)	55 (.03)	71 (.01)	93 (.01)	115	143	191
11	16 (5.0)	19 (2.8)	21 (1.7)	26 (.68)	31 (.34)	38 (.16)	48 (.07)	60 (.03)	76 (.02)	100 (.01)	124	154	206
12	18 (5.5)	20 (3.1)	23 (1.7)	28 (.72)	34 (.34)	41 (.16)	52 (.07)	67 (.03)	82 (.02)	108 (.01)	133	165	220
13	19 (6.2)	21 (3.6)	24 (1.9)	30 (.76)	36 (.40)	43 (.18)	56 (.07)	71 (.04)	87 (.02)	115 (.01)	142	176	235
14	20 (6.7)	23 (3.9)	26 (2.1)	32 (.85)	38 (.45)	46 (.22)	60 (.09)	76 (.04)	93 (.02)	122 (.01)	150	187	249
15	22 (7.0)	24 (4.2)	28 (2.2)	34 (.95)	40 (.45)	49 (.22)	63 (.09)	80 (.04)	98 (.02)	129 (.01)	159	197	264

$(t/\mu) \times 100$ ratios in parentheses are for P(A) = .95 (or more)

TABLE 3b

Table of Sampling Plans for $\beta = 1/2$

c	n												
	$(t/\mu) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	100	50	25	10	5	2.5	1	0.5	0.25	0.1	0.05	0.025	0.01
0	2 (.03)	3 (.02)	4 (.01)	6	8	11	17	23	33	52	73	103	165
1	4 (.52)	5 (.32)	7 (.16)	10 (.07)	13 (.04)	18 (.02)	28 (.01)	40	56	88	124	177	278
2	5 (2.2)	7 (.94)	9 (.54)	13 (.24)	18 (.12)	25 (.05)	39 (.02)	55 (.01)	77 (.01)	120	172	241	381
3	7 (3.3)	9 (1.7)	12 (.85)	17 (.40)	23 (.21)	32 (.10)	49 (.04)	69 (.02)	96 (.01)	153	215	303	478
4	9 (4.1)	11 (2.5)	14 (1.4)	20 (.60)	28 (.30)	38 (.15)	59 (.06)	82 (.03)	115 (.02)	183 (.01)	258	362	571
5	10 (6.4)	13 (3.3)	16 (1.9)	24 (.75)	32 (.40)	45 (.19)	68 (.08)	96 (.04)	134 (.02)	213 (.01)	299	420	663
6	12 (7.2)	14 (4.7)	19 (2.2)	27 (.94)	37 (.46)	51 (.24)	78 (.10)	109 (.05)	155 (.02)	242 (.01)	339	477	753
7	13 (9.7)	16 (5.3)	21 (2.6)	30 (1.1)	41 (.55)	57 (.28)	87 (.11)	122 (.06)	173 (.03)	270 (.01)	379 (.01)	533	841
8	14 (12)	18 (6.0)	23 (3.2)	34 (1.2)	46 (.61)	63 (.33)	96 (.13)	134 (.07)	191 (.03)	298 (.01)	418 (.01)	589	929
9	16 (12)	20 (6.4)	26 (3.3)	37 (1.4)	50 (.73)	69 (.36)	105 (.14)	147 (.08)	208 (.04)	326 (.01)	457 (.01)	643	1,015
10	17 (15)	22 (6.8)	28 (3.7)	40 (1.5)	54 (.78)	77 (.38)	118 (.15)	162 (.08)	226 (.04)	353 (.02)	496 (.01)	698	1,101
11	19 (16)	23 (8.0)	30 (4.1)	43 (1.6)	58 (.83)	83 (.40)	126 (.16)	175 (.08)	244 (.04)	380 (.02)	534 (.01)	752	1,186
12	20 (17)	25 (8.8)	32 (4.6)	47 (1.7)	66 (.87)	89 (.42)	135 (.17)	187 (.09)	261 (.04)	407 (.02)	572 (.01)	805	1,271
13	22 (18)	27 (9.0)	34 (5.0)	50 (1.9)	70 (.90)	95 (.44)	144 (.19)	200 (.09)	278 (.05)	434 (.02)	610 (.01)	858	1,355
14	23 (19)	29 (9.2)	37 (5.0)	53 (2.0)	75 (.92)	101 (.47)	153 (.20)	212 (.10)	295 (.05)	461 (.02)	648 (.01)	911 (.01)	1,438
15	25 (19)	31 (9.4)	39 (5.0)	56 (2.2)	79 (.98)	107 (.49)	162 (.21)	224 (.11)	312 (.05)	488 (.02)	685 (.01)	964 (.01)	1,521

$(t/\mu) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

TABLE 3c

Table of Sampling Plans for $\beta = 1$

c	n												
	$(t/\mu) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	100	50	25	10	5	2.5	1	0.5	0.25	0.1	0.05	0.025	0.01
0	3 (1.7)	5 (1.0)	10 (.51)	24 (.20)	46 (.11)	92 (.06)	231 (.02)	461 (.01)	922	2,303	4,606	9,212	230-2
1	5 (8.0)	9 (4.2)	17 (2.1)	40 (.90)	79 (.45)	158 (.22)	389 (.09)	778 (.05)	1,556 (.02)	3,890 (.01)	7,780	156-2	389-2
2	7 (14)	12 (7.4)	23 (3.7)	55 (1.5)	108 (.76)	216 (.38)	533 (.15)	1,065 (.08)	2,129 (.04)	5,322 (.02)	106-2 (.01)	213-2	532-2
3	9 (19)	15 (10)	29 (5.0)	69 (2.0)	135 (1.0)	271 (.50)	669 (.20)	1,337 (.10)	2,673 (.05)	6,681 (.02)	134-2 (.01)	267-2	668-2
4	11 (22)	19 (12)	34 (6.2)	82 (2.4)	164 (1.2)	324 (.61)	800 (.24)	1,600 (.12)	3,200 (.06)	8,000 (.02)	160-2 (.01)	320-2	800-2
5	13 (25)	22 (13)	40 (7.0)	96 (2.8)	191 (1.4)	376 (.69)	928 (.28)	1,855 (.14)	3,710 (.07)	9,275 (.03)	186-2 (.01	371-2 (.01)	928-2
6	14 (30)	25 (15)	46 (7.5)	109 (3.0)	216 (1.5)	427 (.77)	1,054 (.31)	2,107 (.16)	4,213 (.08)	105-2 (.03)	211-2 (.02)	421-2 (.01)	105-3
7	16 (33)	28 (16)	51 (8.2)	122 (3.3)	242 (1.7)	477 (.83)	1,178 (.34)	2,355 (.17)	4,709 (.08)	118-2 (.03)	235-2 (.02)	471-2 (.01)	118-3
8	18 (35)	31 (17)	57 (9.0)	135 (3.5)	267 (1.8)	527 (.89)	1,300 (.36)	2,600 (.18)	5,200 (.09)	130-2 (.04)	260-2 (.02)	520-2 (.01)	130-3
9	20 (36)	34 (18)	62 (9.3)	147 (3.7)	292 (1.9)	576 (.94)	1,421 (.38)	2,842 (.19)	5,683 (.10)	142-2 (.04)	284-2 (.02)	568-2 (.01)	142-3
10	22 (38)	37 (19)	70 (9.5)	162 (3.9)	316 (2.0)	624 (1.0)	1,541 (.40)	3,082 (.20)	6,163 (.10)	154-2 (.04)	308-2 (.02)	616-2 (.01)	154-3
11	23 (40)	40 (20)	76 (9.8)	175 (4.0)	341 (2.1)	672 (1.1)	1,660 (.42)	3,320 (.21)	6,640 (.10)	166-2 (.04)	332-2 (.02)	664-2 (.01)	166-3
12	25 (42)	43 (20)	81 (10)	187 (4.2)	365 (2.2)	720 (1.1)	1,780 (.43)	3,557 (.22)	7,113 (.11)	178-2 (.04)	336-2 (.02)	711-2 (.01)	178-3
13	27 (42)	45 (21)	86 (10)	200 (4.3)	389 (2.2)	768 (1.1)	1,896 (.45)	3,792 (.22)	7,584 (.11)	190-2 (.05)	379-2 (.02)	758-2 (.01)	190-3
14	29 (44)	48 (22)	91 (11)	212 (4.4)	413 (2.3)	815 (1.1)	2,013 (.46)	4,026 (.23)	8,052 (.12)	201-2 (.05)	403-2 (.02)	805-2 (.01)	201-3
15	31 (44)	51 (22)	97 (11)	224 (4.6)	437 (2.4)	863 (1.2)	2,130 (.47)	4,260 (.24)	8,517 (.12)	213-2 (.05)	426-2 (.02)	852-2 (.01)	213-3

$(t/\mu) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 154-3 = 154,000.

TABLE 3d

Table of Sampling Plans for β = 1 2/3

c	n												
	(t/μ) x 100 Ratio for which P(A) = .10 (or less)												
	100	50	25	15	10	8	5	4	2.5	1.5	1	0.5	0.25
0	3 (9.8)	9 (5.0)	28 (2.5)	66 (1.5)	129 (1.0)	189 (.80)	412 (.50)	591 (.40)	1,280 (.25)	3,031 (.15)	6,061 (.10)	184-2 (.05)	576-2 (.03)
1	6 (22)	16 (12)	48 (5.9)	112 (3.5)	220 (2.3)	319 (1.9)	695 (1.2)	998 (.96)	2,162 (.60)	5,119 (.35)	102-2 (.23)	311-2 (.11)	973-2 (.06)
2	8 (31)	22 (16)	66 (8.1)	155 (4.8)	301 (3.2)	437 (2.6)	951 (1.6)	1,365 (1.3)	2,957 (.81)	7,003 (.49)	140-2 (.32)	426-2 (.16)	133-3 (.08)
3	10 (38)	28 (19)	83 (9.6)	194 (5.7)	378 (3.8)	548 (3.1)	1,194 (1.9)	1,714 (1.5)	3,712 (.96)	8,791 (.58)	176-2 (.38)	534-2 (.19)	167-3 (.09)
4	12 (42)	33 (21)	100 (11)	232 (6.4)	452 (4.3)	656 (3.4)	1,428 (2.1)	2,050 (1.7)	4,442 (1.1)	105-2 (.65)	210-2 (.42)	640-2 (.21)	200-3 (.10)
5	14 (46)	39 (23)	116 (12)	269 (6.9)	525 (4.6)	761 (3.7)	1,657 (2.3)	2,379 (1.9)	5,153 (1.2)	122-2 (.69)	244-2 (.45)	742-2 (.23)	232-3 (.11)
6	17 (47)	44 (25)	132 (12)	306 (7.4)	596 (4.9)	864 (3.9)	1,881 (2.5)	2,701 (2.0)	5,852 (1.3)	139-2 (.75)	277-2 (.48)	843-2 (.25)	263-3 (.12)
7	19 (50)	49 (26)	148 (13)	342 (7.7)	666 (5.2)	965 (4.2)	2,102 (2.6)	3,019 (2.1)	6,540 (1.3)	155-2 (.78)	310-2 (.51)	942-2 (.26)	294-3 (.13)
8	21 (52)	54 (27)	165 (13)	377 (8.1)	735 (5.4)	1,066 (4.3)	2,321 (2.7)	3,333 (2.2)	7,220 (1.4)	171-2 (.80)	342-2 (.53)	104-3 (.27)	325-3 (.13)
9	23 (54)	59 (28)	181 (14)	412 (8.3)	803 (5.6)	1,165 (4.5)	2,537 (2.8)	3,643 (2.2)	7,893 (1.4)	187-2 (.84)	374-2 (.55)	114-3 (.28)	355-3 (.14)
10	24 (57)	68 (28)	196 (14)	447 (8.6)	871 (5.7)	1,263 (4.6)	2,752 (2.9)	3,951 (2.3)	8,560 (1.5)	203-2 (.86)	405-2 (.57)	123-3 (.29)	385-3 (.14)
11	26 (59)	73 (29)	211 (15)	482 (8.9)	938 (5.9)	1,361 (4.7)	2,964 (2.9)	4,256 (2.3)	9,222 (1.5)	218-2 (.89)	437-2 (.53)	133-3 (.30)	415-3 (.15)
12	28 (60)	78 (29)	226 (15)	516 (9.0)	1,005 (6.0)	1,458 (4.8)	3,176 (3.0)	4,560 (2.4)	9,879 (1.5)	234-2 (.90)	468-2 (.59)	142-3 (.30)	445-3 (.15)
13	30 (61)	83 (29)	241 (15)	550 (9.1)	1,072 (6.1)	1,554 (4.9)	3,386 (3.1)	4,862 (2.5)	105-2 (1.6)	249-2 (.92)	499-2 (.61)	152-3 (.31)	475-3 (.15)
14	33 (61)	88 (30)	256 (15)	584 (9.3)	1,138 (6.3)	1,650 (5.0)	3,595 (3.1)	5,162 (2.5)	112-2 (1.6)	265-2 (.94)	530-2 (.62)	161-3 (.31)	505-3 (.16)
15	35 (62)	93 (30)	270 (16)	618 (9.4)	1,203 (6.4)	1,746 (5.1)	3,803 (3.2)	5,460 (2.6)	118-2 (.16)	280-2 (.95)	560-2 (.63)	170-3 (.32)	535-3 (.16)

(t/μ) x 100 ratios in parentheses are for P(A) = .95 (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 218-2 = 21,800.

TABLE 3e

Table of Sampling Plans for $\beta = 2$

c	n												
	$(t/\mu) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	100	50	25	15	12	10	8	5	4	2.5	1.5	1	0.5
0	3 (15)	12 (7.4)	47 (3.7)	131 (2.2)	206 (1.8)	296 (1.5)	461 (1.2)	1,152 (.76)	1,772 (.62)	4,700 (.38)	128-2 (.23)	288-2 (.16)	115-3 (.08)
1	6 (29)	21 (15)	80 (7.5)	223 (4.5)	348 (3.6)	499 (3.0)	778 (2.4)	1,945 (1.5)	2,993 (1.2)	7,939 (.76)	216-2 (.46)	486-2 (.30)	194-3 (.15)
2	8 (38)	29 (19)	110 (9.8)	305 (5.8)	476 (4.7)	683 (3.9)	1,064 (3.1)	2,661 (2.0)	4,094 (1.6)	109-2 (.98)	296-2 (.60)	665-2 (.40)	266-3 (.20)
3	11 (43)	36 (22)	138 (11)	382 (6.7)	597 (5.4)	857 (4.5)	1,336 (3.6)	3,341 (2.3)	5,140 (1.8)	136-2 (1.1)	371-2 (.70)	835-2 (.47)	334-3 (.24)
4	13 (48)	43 (24)	167 (12)	457 (7.4)	714 (5.9)	1,025 (5.0)	1,599 (4.0)	3,977 (2.5)	6,150 (2.0)	163-2 (1.2)	444-2 (.76)	999-2 (.51)	400-3 (.26)
5	15 (52)	50 (26)	194 (13)	530 (7.9)	829 (6.3)	1,190 (5.3)	1,855 (4.2)	4,638 (2.7)	7,135 (2.2)	189-2 (1.3)	515-2 (.81)	116-3 (.54)	464-3 (.28)
6	17 (55)	57 (28)	220 (14)	602 (8.3)	941 (6.6)	1,351 (5.6)	2,106 (4.5)	5,214 (2.8)	8,102 (2.3)	215-2 (1.4)	585-2 (.85)	132-3 (.57)	527-3 (.29)
7	19 (58)	64 (29)	245 (14)	673 (8.7)	1,051 (6.9)	1,510 (5.8)	2,354 (4.6)	5,886 (2.9)	9,055 (2.4)	240-2 (1.5)	654-2 (.88)	147-3 (.60)	589-3 (.30)
8	21 (60)	71 (30)	272 (15)	743 (9.0)	1,161 (7.2)	1,667 (6.0)	2,600 (4.8)	6,498 (3.0)	9,997 (2.4)	265-2 (1.5)	722-2 (.91)	162-3 (.62)	650-3 (.31)
9	23 (62)	77 (30)	297 (15)	812 (9.2)	1,274 (7.4)	1,822 (6.1)	2,841 (4.9)	7,103 (3.1)	109-2 (2.5)	290-2 (1.5)	789-2 (.94)	178-3 (.63)	710-3 (.32)
10	26 (62)	87 (31)	322 (15)	881 (9.4)	1,376 (7.5)	1,976 (6.3)	3,081 (5.0)	7,704 (3.2)	119-2 (2.6)	314-2 (1.6)	856-2 (.96)	193-3 (.65)	770-3 (.33)
11	28 (63)	94 (31)	347 (16)	949 (9.6)	1,482 (7.7)	2,128 (6.4)	3,320 (5.1)	8,299 (3.3)	128-2 (2.6)	339-2 (1.6)	922-2 (9.8)	207-3 (.66)	830-3 (.34)
12	30 (64)	100 (32)	372 (16)	1,017 (9.8)	1,588 (7.8)	2,280 (6.5)	3,556 (5.2)	8,891 (3.3)	137-2 (2.7)	363-2 (1.6)	988-2 (1.0)	222-3 (.67)	889-3 (.34)
13	32 (65)	107 (32)	396 (16)	1,084 (10)	1,693 (8.0)	2,431 (6.6)	3,792 (5.3)	9,479 (3.4)	146-2 (2.7)	387-2 (1.7)	105-3 (1.0)	237-3 (.68)	948-3 (.35)
14	34 (66)	113 (33)	421 (17)	1,151 (10)	1,798 (8.1)	2,581 (6.7)	4,026 (5.4)	101-2 (3.4)	155-2 (2.8)	411-2 (1.7)	112-3 (1.0)	252-3 (.69)	101-4 (.35)
15	36 (67)	120 (33)	445 (17)	1,217 (10)	1,902 (8.2)	2,730 (6.8)	4,258 (5.5)	106-2 (3.5)	164-2 (2.8)	435-2 (1.7)	118-3 (1.0)	266-3 (.70)	106-4 (.36)

$(t/\mu) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 118-3 = 118,000.

TABLE 3f

Table of Sampling Plans for $\beta = 2\ 1/2$

c	n												
	$(t/\mu) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	100	50	40	25	15	12	10	8	6.5	5	4	2.5	1.5
0	4 (20)	20 (10)	31 (8.7)	100 (5.4)	360 (3.3)	623 (2.6)	1,002 (2.2)	1,772 (1.7)	2,879 (1.4)	5,618 (1.1)	9,596 (.88)	320-2 (.55)	115-3 (.33)
1	6 (37)	34 (18)	53 (15)	170 (9.6)	608 (5.7)	1,052 (4.6)	1,692 (3.8)	2,993 (3.0)	4,863 (2.5)	9,488 (1.9)	162-2 (1.5)	540-2 (.96)	194-3 (.58)
2	9 (45)	46 (23)	72 (19)	233 (12)	832 (7.0)	1,439 (5.6)	2,314 (4.7)	4,094 (3.7)	6,653 (3.1)	130-2 (2.3)	222-2 (1.9)	739-2 (1.2)	266-3 (.71)
3	11 (51)	58 (25)	91 (21)	292 (13)	1,044 (7.8)	1,806 (6.3)	2,905 (5.2)	5,140 (4.2)	8,352 (3.4)	163-2 (2.6)	278-2 (2.1)	928-2 (1.3)	334-3 (.80)
4	13 (56)	70 (27)	109 (23)	350 (14)	1,250 (8.5)	2,161 (6.8)	3,476 (5.6)	6,150 (4.5)	9,993 (3.7)	195-2 (2.8)	333-2 (2.3)	111-3 (1.4)	400-3 (.86)
5	16 (58)	81 (29)	127 (24)	406 (15)	1,450 (9.0)	2,507 (7.2)	4,033 (6.0)	7,135 (4.7)	116-2 (3.9)	226-2 (3.0)	386-2 (2.4)	129-3 (1.5)	464-3 (.91)
6	18 (61)	92 (30)	144 (25)	460 (16)	1,646 (9.4)	2,847 (7.5)	4,580 (6.2)	8,102 (4.9)	132-2 (4.1)	257-2 (3.1)	439-2 (2.5)	146-3 (1.6)	527-3 (.94)
7	20 (64)	103 (31)	163 (25)	515 (16)	1,840 (9.7)	3,182 (7.7)	5,118 (6.4)	9,055 (5.1)	147-2 (4.2)	287-2 (3.2)	490-2 (2.6)	163-3 (1.6)	589-3 (.97)
8	22 (66)	113 (32)	180 (26)	568 (17)	2,031 (10)	3,513 (8.0)	5,650 (6.6)	9,997 (5.2)	162-2 (4.3)	317-2 (3.3)	541-2 (2.7)	180-3 (1.6)	650-3 (1.0)
9	25 (67)	124 (32)	197 (27)	621 (17)	2,220 (10)	3,840 (8.1)	6,177 (6.7)	109-2 (5.3)	178-2 (4.4)	346-2 (3.4)	592-2 (2.7)	197-3 (1.7)	710-3 (1.0)
10	27 (68)	137 (33)	214 (27)	673 (17)	2,408 (10)	4,164 (8.3)	6,699 (6.8)	119-2 (5.4)	193-2 (4.5)	376-2 (3.4)	642-2 (2.8)	214-3 (1.7)	770-3 (1.0)
11	29 (69)	148 (33)	230 (28)	725 (18)	2,594 (11)	4,486 (8.4)	7,217 (7.0)	128-2 (5.5)	207-2 (4.6)	405-2 (3.5)	692-2 (2.8)	231-3 (1.7)	830-3 (1.1)
12	31 (70)	158 (34)	246 (28)	777 (18)	2,779 (11)	4,806 (8.6)	7,732 (7.1)	137-2 (5.6)	222-2 (4.6)	434-2 (3.5)	741-2 (2.9)	247-3 (1.8)	889-3 (1.1)
13	33 (71)	168 (34)	263 (29)	828 (18)	2,963 (11)	5,124 (8.7)	8,243 (7.1)	146-2 (5.7)	237-2 (4.7)	462-2 (3.6)	790-2 (2.9)	263-3 (1.8)	948-3 (1.1)
14	35 (72)	179 (35)	279 (29)	879 (18)	3,145 (11)	5,440 (8.8)	8,752 (7.2)	155-2 (5.8)	252-2 (4.7)	491-2 (3.6)	839-2 (2.9)	280-3 (1.8)	101-4 (1.1)
15	37 (73)	189 (36)	295 (29)	930 (19)	3,327 (11)	5,755 (8.8)	9,258 (7.3)	164-2 (5.8)	266-2 (4.8)	519-2 (3.7)	887-2 (3.0)	296-3 (1.8)	106-4 (1.1)

$(t/\mu) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 296-3 = 296,000.

TABLE 38

Table of Sampling Plans for β = 3 1/3

c	n													
	(t/μ) x 100 Ratio for which P(A) = .10 (or less)													
	100	65	50	40	30	25	20	15	12	10	8	6.5	5
0	4 (30)	14 (21)	34 (16)	70 (13)	183 (9.6)	334 (8.0)	698 (6.4)	1,77[illegible] (4.8)	3,839 (3.8)	6,979 (3.2)	149-2 (2.5)	288-2 (2.1)	698-2 (1.6)
1	7 (46)	24 (32)	57 (24)	119 (19)	309 (14)	564 (12)	1,179 (9.8)	2,993 (7.4)	6,484 (5.8)	118-2 (4.8)	251-2 (3.9)	486-2 (3.2)	118-3 (2.4)
2	9 (56)	33 (37)	78 (28)	164 (23)	423 (17)	772 (14)	1,613 (11)	4,094 (8.7)	8,870 (6.8)	161-2 (5.7)	343-2 (4.5)	665-2 (3.7)	161-3 (2.8)
3	12 (60)	42 (40)	98 (31)	206 (25)	531 (19)	969 (15)	2,025 (12)	5,140 (9.4)	111-2 (7.5)	202-2 (6.2)	431-2 (4.9)	835-2 (4.0)	202-3 (3.1)
4	14 (65)	51 (42)	117 (33)	246 (26)	635 (20)	1,159 (16)	2,423 (13)	6,150 (10)	133-2 ([illegible])	242-2 (6.6)	516-2 (5.2)	999-2 (4.3)	242-3 (3.3)
5	16 (68)	59 (44)	136 (34)	286 (27)	737 (21)	1,345 (17)	2,811 (14)	7,135 (10)	[illegible] (8.2)	281-2 (6.8)	598-2 (5.4)	116-3 (4.4)	281-3 (3.4)
6	19 (69)	67 (46)	157 (35)	325 (28)	836 (21)	1,527 (17)	3,192 (14)	8,102 (11)	176-2 (8.4)	319-2 (7.0)	679-2 (5.6)	132-3 (4.6)	319-3 (3.5)
7	21 (71)	75 (47)	176 (36)	363 (29)	935 (22)	1,698 (18)	3,567 (14)	9,055 (11)	196-2 (8.7)	357-2 (7.2)	759-2 (5.8)	147-3 (4.7)	357-3 (3.6)
8	23 (73)	83 (47)	194 (36)	400 (29)	1,032 (22)	1,884 (18)	3,938 (15)	9,997 (11)	217-2 (8.8)	394-2 (7.4)	838-2 (5.9)	162-3 (4.8)	394-3 (3.7)
9	26 (74)	90 (48)	212 (37)	438 (30)	1,1[illegible]8 (22)	2,059 (19)	4,305 (15)	109-2 (11)	237-2 (9.0)	430-2 (7.5)	917-2 (6.0)	178-3 (4.9)	430-3 (3.7)
10	28 (75)	101 (48)	230 (38)	475 (30)	1,228 (23)	2,233 (19)	4,669 (15)	119-2 (12)	257-2 (9.2)	467-2 (7.6)	994-2 (6.0)	193-3 (4.9)	467-3 (3.8)
11	30 (76)	108 (49)	247 (38)	511 (31)	1,318 (23)	2,406 (19)	5,030 (15)	128-2 (12)	277-2 (9.3)	503-2 (7.7)	107-3 (6.1)	207-3 (5.0)	503-3 (3.8)
12	32 (77)	116 (49)	265 (39)	548 (31)	1,412 (23)	2,578 (20)	5,389 (15)	137-2 (12)	296-2 (9.4)	539-2 (7.8)	115-3 (6.2)	222-3 (5.0)	539-3 (3.9)
13	35 (77)	124 (50)	283 (39)	584 (31)	1,505 (24)	2,748 (20)	5,745 (16)	146-2 (12)	316-2 (9.4)	574-2 (7.9)	122-3 (6.2)	237-3 (5.1)	574-3 (3.9)
14	37 (78)	131 (50)	300 (39)	620 (32)	1,598 (24)	2,918 (20)	6,100 (16)	155-2 (12)	335-2 (9.5)	610-2 (8.0)	130-3 (6.3)	252-3 (5.2)	610-3 (3.9)
15	39 (79)	139 (51)	317 (40)	656 (32)	1,690 (24)	3,086 (20)	6,453 (16)	164-2 (12)	355-2 (9.6)	645-2 (8.0)	137-3 (6.4)	266-3 (5.2)	645-3 (4.0)

(t/μ) x 100 ratios in parentheses are for P(A) = .95 (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 319-3 = 319,000.

TABLE 3h

Table of Sampling Plans for β = 4

c	n												
	(t/μ) x 100 Ratio for which P(A) = .10 (or less)												
	100	80	65	50	40	30	25	20	15	12	10	8	6.5
0	4 (37)	9 (30)	20 (25)	55 (19)	134 (15)	419 (12)	886 (9.4)	2,094 (7.6)	6,774 (5.8)	164-2 (4.7)	329-2 (4.0)	768-2 (3.2)	177-3 (2.6)
1	7 (53)	15 (44)	33 (36)	93 (27)	228 (22)	708 (16)	1,497 (14)	3,537 (11)	114-2 (8.0)	278-2 (6.5)	556-2 (5.6)	130-3 (4.5)	299-3 (3.7)
2	9 (62)	21 (50)	46 (40)	128 (31)	312 (25)	968 (19)	2,047 (16)	4,839 (12)	157-2 (9.2)	380-2 (7.4)	760-2 (6.2)	177-3 (5.1)	409-3 (4.2)
3	12 (66)	26 (54)	57 (44)	162 (33)	391 (27)	1,215 (20)	2,570 (17)	6,074 (13)	197-2 (10)	477-2 (7.9)	954-2 (6.6)	223-3 (5.5)	514-3 (4.5)
4	15 (68)	31 (56)	69 (46)	194 (35)	468 (28)	1,454 (21)	3,075 (18)	7,268 (14)	235-2 (10)	571-2 (8.3)	114-3 (7.0)	266-3 (5.8)	615-3 (4.7)
5	17 (71)	37 (58)	80 (48)	225 (36)	543 (29)	1,687 (22)	3,568 (18)	8,432 (15)	273-2 (11)	663-2 (8.6)	133-3 (7.2)	309-3 (6.0)	713-3 (4.9)
6	19 (74)	42 (59)	91 (49)	255 (37)	616 (30)	1,915 (22)	4,051 (19)	9,575 (15)	310-2 (11)	752-2 (8.8)	150-3 (7.4)	351-3 (6.1)	810-3 (5.0)
7	22 (75)	47 (60)	102 (50)	286 (38)	689 (30)	2,140 (23)	4,528 (19)	107-2 (15)	346-2 (11)	841-2 (9.0)	168-3 (7.6)	392-3 (6.2)	905-3 (5.1)
8	24 (76)	52 (61)	112 (50)	315 (39)	760 (31)	2,363 (23)	4,998 (19)	118-2 (15)	382-2 (12)	928-2 (9.1)	186-3 (7.7)	433-3 (6.3)	100-4 (5.2)
9	27 (77)	57 (62)	123 (51)	344 (39)	831 (31)	2,583 (23)	5,464 (20)	129-2 (16)	418-2 (12)	101-3 (9.2)	203-3 (7.8)	474-3 (6.3)	109-4 (5.3)
10	29 (78)	64 (62)	136 (51)	373 (40)	901 (32)	2,802 (24)	5,926 (20)	140-2 (16)	453-2 (12)	110-3 (9.4)	220-3 (7.8)	514-3 (6.4)	119-4 (5.3)
11	31 (79)	69 (63)	147 (52)	402 (40)	971 (32)	3,018 (24)	6,384 (20)	151-2 (16)	488-2 (12)	119-3 (9.4)	237-3 (7.9)	553-3 (6.4)	128-4 (5.4)
12	33 (80)	74 (63)	157 (52)	431 (40)	1,040 (32)	3,233 (24)	6,840 (20)	162-2 (16)	523-2 (12)	127-3 (9.5)	254-3 (8.0)	593-3 (6.5)	137-4 (5.4)
13	36 (81)	79 (64)	167 (53)	460 (41)	1,109 (33)	3,447 (24)	7,292 (20)	172-2 (16)	558-2 (12)	135-3 (9.6)	271-3 (8.1)	632-3 (6.5)	146-4 (5.5)
14	38 (82)	84 (64)	178 (53)	488 (41)	1,177 (33)	3,660 (25)	7,742 (21)	183-2 (17)	592-2 (12)	144-3 (9.7)	288-3 (8.2)	671-3 (6.6)	155-4 (5.5)
15	40 (82)	89 (64)	188 (53)	516 (41)	1,246 (33)	3,872 (25)	8,190 (21)	194-2 (17)	626-2 (12)	152-3 (9.8)	304-3 (8.2)	710-3 (6.6)	164-4 (5.6)

(t/μ) x 100 ratios in parentheses are for P(A) = .95 (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 304-3 = 304,000.

TABLE 3g

Table of Sampling Plans for β = 3 1/3

c	n												
	$(t/\mu) \times 100$ Ratio for which P(A) = .10 (or less)												
	100	65	50	40	30	25	20	15	12	10	8	6.5	5
0	4 (30)	14 (21)	34 (16)	70 (13)	183 (9.6)	334 (8.0)	698 (6.4)	1,772 (4.8)	3,839 (3.8)	6,979 (3.2)	149-2 (2.5)	288-2 (2.1)	698-2 (1.6)
1	7 (46)	24 (32)	57 (24)	119 (19)	309 (14)	564 (12)	1,179 (9.8)	2,993 (7.4)	6,484 (5.8)	118-2 (4.8)	251-2 (3.9)	486-2 (3.2)	118-3 (2.4)
2	9 (56)	33 (37)	78 (28)	164 (23)	423 (17)	772 (14)	1,613 (11)	4,094 (8.7)	8,870 (6.8)	161-2 (5.7)	343-2 (4.5)	665-2 (3.7)	161-3 (2.8)
3	12 (60)	42 (40)	98 (31)	206 (25)	531 (19)	969 (15)	2,025 (12)	5,140 (9.4)	111-2 (7.5)	202-2 (6.2)	431-2 (4.9)	835-2 (4.0)	202-3 (3.1)
4	14 (65)	51 (42)	117 (33)	246 (26)	635 (20)	1,159 (16)	2,423 (13)	6,150 (10)	133-2 (7.9)	242-2 (6.6)	516-2 (5.2)	999-2 (4.3)	242-3 (3.3)
5	16 (68)	59 (44)	136 (34)	286 (27)	737 (21)	1,345 (17)	2,811 (14)	7,135 (10)	155-2 (8.2)	281-2 (6.8)	598-2 (5.4)	116-3 (4.4)	281-3 (3.4)
6	19 (69)	67 (46)	157 (35)	325 (28)	836 (21)	1,527 (17)	3,192 (14)	8,102 (11)	176-2 (8.4)	319-2 (7.0)	679-2 (5.6)	132-3 (4.6)	319-3 (3.5)
7	21 (71)	75 (47)	176 (36)	363 (29)	935 (22)	1,698 (18)	3,567 (14)	9,055 (11)	196-2 (8.7)	357-2 (7.2)	759-2 (5.8)	147-3 (4.7)	357-3 (3.6)
8	23 (73)	83 (47)	194 (36)	400 (29)	1,032 (22)	1,884 (18)	3,938 (15)	9,997 (11)	217-2 (8.8)	394-2 (7.4)	838-2 (5.9)	162-3 (4.8)	394-3 (3.7)
9	26 (74)	90 (48)	212 (37)	438 (30)	1,128 (22)	2,059 (19)	4,305 (15)	109-2 (11)	237-2 (9.0)	430-2 (7.5)	917-2 (6.0)	178-3 (4.9)	430-3 (3.7)
10	28 (75)	101 (48)	230 (38)	475 (30)	1,228 (23)	2,233 (19)	4,669 (15)	119-2 (12)	257-2 (9.2)	467-2 (7.6)	994-2 (6.0)	193-3 (4.9)	467-3 (3.8)
11	30 (76)	108 (49)	247 (38)	511 (31)	1,318 (23)	2,406 (19)	5,030 (15)	128-2 (12)	277-2 (9.3)	503-2 (7.7)	107-3 (6.1)	207-3 (5.0)	503-3 (3.8)
12	32 (77)	116 (49)	265 (39)	548 (31)	1,412 (23)	2,578 (20)	5,389 (15)	137-2 (12)	296-2 (9.4)	539-2 (7.8)	115-3 (6.2)	222-3 (5.0)	539-3 (3.9)
13	35 (77)	124 (50)	283 (39)	584 (31)	1,505 (24)	2,748 (20)	5,745 (16)	146-2 (12)	316-2 (9.4)	574-2 (7.9)	122-3 (6.2)	237-3 (5.1)	574-3 (3.9)
14	37 (78)	131 (50)	300 (39)	620 (32)	1,598 (24)	2,918 (20)	6,100 (16)	155-2 (12)	335-2 (9.5)	610-2 (8.0)	130-3 (6.3)	252-3 (5.2)	610-3 (3.9)
15	39 (79)	139 (51)	317 (40)	656 (32)	1,690 (24)	3,086 (20)	6,453 (16)	164-2 (12)	355-2 (9.6)	645-2 (8.0)	137-3 (6.4)	266-3 (5.2)	645-3 (4.0)

$(t/\mu) \times 100$ ratios in parentheses are for P(A) = .95 (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 319-3 = 319,000.

TABLE 3h

Table of Sampling Plans for $\beta = 4$

c	n												
	$(t/\mu) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	100	80	65	50	40	30	25	20	15	12	10	8	6.5
0	4 (37)	9 (30)	20 (25)	55 (19)	134 (15)	419 (12)	886 (9.4)	2,094 (7.6)	6,774 (5.8)	164-2 (4.7)	329-2 (4.0)	768-2 (3.2)	177-3 (2.6)
1	7 (53)	15 (44)	33 (36)	93 (27)	228 (22)	708 (16)	1,497 (14)	3,537 (11)	114-2 (8.0)	278-2 (6.5)	556-2 (5.6)	130-3 (4.5)	299-3 (3.7)
2	9 (62)	21 (50)	46 (40)	128 (31)	312 (25)	968 (19)	2,047 (16)	4,839 (12)	157-2 (9.2)	380-2 (7.4)	760-2 (6.2)	177-3 (5.1)	409-3 (4.2)
3	12 (66)	26 (54)	57 (44)	162 (33)	391 (27)	1,215 (20)	2,570 (17)	6,074 (13)	197-2 (10)	477-2 (7.9)	954-2 (6.6)	223-3 (5.5)	514-3 (4.5)
4	15 (68)	31 (56)	69 (46)	194 (35)	468 (28)	1,454 (21)	3,075 (18)	7,268 (14)	235-2 (10)	571-2 (8.3)	114-3 (7.0)	266-3 (5.8)	615-3 (4.7)
5	17 (71)	37 (58)	80 (48)	225 (36)	543 (29)	1,687 (22)	3,568 (18)	8,432 (15)	273-2 (11)	663-2 (8.6)	133-3 (7.2)	309-3 (6.0)	713-3 (4.9)
6	19 (74)	42 (59)	91 (49)	255 (37)	616 (30)	1,915 (22)	4,051 (19)	9,575 (15)	310-2 (11)	752-2 (8.8)	150-3 (7.4)	351-3 (6.1)	810-3 (5.0)
7	22 (75)	47 (60)	102 (50)	286 (38)	689 (30)	2,140 (23)	4,528 (19)	107-2 (15)	346-2 (11)	841-2 (9.0)	168-3 (7.6)	392-3 (6.2)	905-3 (5.1)
8	24 (76)	52 (61)	112 (50)	315 (39)	760 (31)	2,363 (23)	4,998 (19)	118-2 (15)	382-2 (12)	928-2 (9.1)	186-3 (7.7)	433-3 (6.3)	100-4 (5.2)
9	27 (77)	57 (62)	123 (51)	344 (39)	831 (31)	2,583 (23)	5,464 (20)	129-2 (16)	418-2 (12)	101-3 (9.2)	203-3 (7.8)	474-3 (6.3)	109-4 (5.3)
10	29 (78)	64 (62)	136 (51)	373 (40)	901 (32)	2,802 (24)	5,926 (20)	140-2 (16)	453-2 (12)	110-3 (9.4)	220-3 (7.8)	514-3 (6.4)	119-4 (5.3)
11	31 (79)	69 (63)	147 (52)	402 (40)	971 (32)	3,018 (24)	6,384 (20)	151-2 (16)	488-2 (12)	119-3 (9.4)	237-3 (7.9)	553-3 (6.4)	128-4 (5.4)
12	33 (80)	74 (63)	157 (52)	431 (40)	1,040 (32)	3,233 (24)	6,840 (20)	162-2 (16)	523-2 (12)	127-3 (9.5)	254-3 (8.0)	593-3 (6.5)	137-4 (5.4)
13	36 (81)	79 (64)	167 (53)	460 (41)	1,109 (33)	3,447 (24)	7,292 (20)	172-2 (16)	558-2 (12)	135-3 (9.6)	271-3 (8.1)	632-3 (6.5)	146-4 (5.5)
14	38 (82)	84 (64)	178 (53)	488 (41)	1,177 (33)	3,660 (25)	7,742 (21)	183-2 (17)	592-2 (12)	144-3 (9.7)	288-3 (8.2)	671-3 (6.6)	155-4 (5.5)
15	40 (82)	89 (64)	188 (53)	516 (41)	1,246 (33)	3,872 (25)	8,190 (21)	194-2 (17)	626-2 (12)	152-3 (9.8)	304-3 (8.2)	710-3 (6.6)	164-4 (5.6)

$(t/\mu) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 304-3 = 304,000.

TABLE 31

Table of Sampling Plans for β = 5

c	n												
	(t/μ) x 100 Ratio for which P(A) = .10 (or less)												
	100	80	65	50	45	40	35	30	25	20	15	12	10
0	4 (46)	11 (37)	31 (30)	113 (23)	192 (21)	344 (19)	678 (16)	1,440 (14)	3,599 (12)	110-2 (9.4)	461-2 (7.0)	135-3 (5.6)	329-3 (4.8)
1	7 (53)	19 (49)	53 (40)	193 (31)	325 (28)	581 (25)	1,145 (22)	2,432 (19)	6,079 (15)	185-2 (12)	778-2 (9.2)	229-3 (7.5)	556-3 (6.3)
2	10 (68)	26 (55)	72 (45)	264 (34)	444 (31)	795 (27)	1,566 (24)	3,327 (21)	8,316 (17)	253-2 (14)	106-3 (10)	313-3 (8.3)	760-3 (7.0)
3	12 (73)	33 (58)	90 (47)	331 (36)	557 (33)	998 (29)	1,965 (25)	4,176 (22)	104-2 (18)	318-2 (14)	134-3 (11)	393-3 (8.8)	954-3 (7.4)
4	15 (76)	40 (60)	108 (49)	396 (38)	667 (34)	1,194 (30)	2,352 (26)	4,997 (23)	125-2 (19)	381-2 (15)	160-3 (11)	470-3 (9.1)	114-4 (7.6)
5	17 (78)	46 (62)	125 (51)	460 (39)	773 (35)	1,385 (31)	2,728 (27)	5,797 (23)	145-2 (19)	442-2 (15)	186-3 (12)	546-3 (9.4)	132-4 (7.8)
6	20 (79)	53 (63)	143 (52)	522 (40)	878 (36)	1,572 (32)	3,098 (28)	6,583 (24)	165-2 (20)	502-2 (16)	211-3 (12)	620-3 (9.6)	150-4 (8.0)
7	22 (80)	59 (64)	162 (52)	583 (40)	981 (36)	1,757 (32)	3,463 (28)	7,357 (24)	184-2 (20)	561-2 (16)	235-3 (12)	692-3 (9.7)	168-4 (8.1)
8	25 (81)	66 (65)	179 (53)	644 (41)	1,083 (37)	1,940 (33)	3,823 (29)	8,122 (25)	203-2 (20)	619-2 (16)	260-3 (12)	764-3 (9.8)	187-4 (8.2)
9	27 (82)	72 (66)	195 (54)	704 (41)	1,184 (37)	2,121 (33)	4,179 (29)	8,879 (25)	222-2 (20)	676-2 (16)	284-3 (12)	836-3 (10)	203-4 (8.3)
10	30 (83)	81 (66)	212 (54)	763 (41)	1,284 (37)	2,300 (33)	4,532 (29)	9,630 (25)	241-2 (21)	734-2 (17)	308-3 (12)	906-3 (10)	220-4 (8.4)
11	32 (84)	87 (66)	228 (55)	822 (42)	1,384 (38)	2,478 (34)	4,882 (29)	104-2 (25)	259-2 (21)	790-2 (17)	332-3 (13)	976-3 (10)	237-4 (8.5)
12	34 (84)	93 (67)	244 (55)	881 (42)	1,482 (38)	2,655 (34)	5,230 (30)	111-2 (25)	278-2 (21)	847-2 (17)	356-3 (13)	105-4 (10)	254-4 (8.6)
13	37 (85)	99 (67)	261 (55)	939 (42)	1,580 (38)	2,830 (34)	5,576 (30)	118-2 (26)	296-2 (21)	903-2 (17)	379-3 (13)	112-4 (10)	271-4 (8.6)
14	39 (85)	105 (68)	277 (56)	997 (43)	1,678 (39)	3,005 (34)	5,920 (30)	126-2 (26)	314-2 (21)	958-2 (17)	403-3 (13)	118-4 (10)	288-4 (8.6)
15	41 (86)	111 (68)	293 (56)	1,055 (43)	1,775 (39)	3,178 (35)	6,263 (30)	133-2 (26)	333-2 (21)	101-3 (17)	426-3 (13)	125-4 (10)	304-4 (8.7)

(t/μ) x 100 ratios in parentheses are for P(A) = .95 (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 203-2 = 20,300.

TABLE 4

Table of Mean Life Multipliers

Approximate Values for $\mu_{.95}/\mu_{.10}$

c	β								
	1/3	1/2	1	1 2/3	2	2 1/2	3 1/3	4	5
0			45	10	6.7	4.6	3.1	2.6	2.2
1	2000	150	11	4.3	3.3	2.6	2.1	1.8	1.6
2	325	45	6.7	3.1	2.6	2.1	1.8	1.6	1.5
3	140	25	5.0	2.6	2.2	1.9	1.6	1.5	1.4
4	75	17	4.1	2.3	2.0	1.8	1.5	1.4	1.3
5	50	13	3.6	2.2	1.9	1.7	1.5	1.4	1.3
6	35	11	3.2	2.0	1.8	1.6	1.4	1.3	1.3
7	27	9.1	3.0	1.9	1.7	1.6	1.4	1.3	1.3
8	23	8.0	2.8	1.9	1.7	1.5	1.4	1.3	1.2
9	20	7.0	2.7	1.8	1.6	1.5	1.3	1.3	1.2
10	18	6.4	2.5	1.8	1.6	1.5	1.3	1.3	1.2
11	16	6.0	2.4	1.7	1.6	1.4	1.3	1.3	1.2
12	14	5.6	2.3	1.7	1.5	1.4	1.3	1.2	1.2
13	13	5.2	2.2	1.6	1.5	1.4	1.3	1.2	1.2
14	12	5.0	2.2	1.6	1.5	1.4	1.3	1.2	1.2
15	11	4.8	2.1	1.6	1.5	1.4	1.3	1.2	1.2

TABLE 5a

Table of (t/μ)x100 Ratios for β = 1/2 for the 105B Plans

Sample Size Code Letter	Acceptable Quality Level (AQL)													
	p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	(t/μ) x 100	11x10^{-7}	61x10^{-7}	21x10^{-6}	50x10^{-6}	11x10^{-5}	31x10^{-5}	80x10^{-5}	21x10^{-4}	51x10^{-4}	11x10^{-3}	32x10^{-3}	83x10^{-3}	.23	.56
A	(t/μ)x100 at LTPD Quality Level [P(A) = .10]													↓	66
B													↓	29	↑
C												↓	10	↑	↓
D											↓	5.4	↑	↓	19
E										↓	2.7	↑	↓	8.4	18
F									↓	1.1	↑	↓	3.6	7.3	13
G								↓	.42	↑	↓	1.3	2.4	4.0	8.6
H							↓	.22	↑	↓	.64	1.2	2.0	4.0	7.1
I						↓	.11	↑	↓	.31	.59	.96	1.4	2.5	4.9
J					↓	.048	↑	↓	.14	.26	.42	.60	1.0	2.0	4.2
K				↓	.022	↑	↓	.064	.12	.19	.28	.48	.74	1.5	3.3
L			↓	.012	↑	↓	.034	.065	.10	.15	.20	.40	.69	1.4	2.8
M		↓	.005	↑	↓	.016	.029	.047	.066	.089	.18	.29	.62	1.1	2.2
N		.003	↑	↓	.009	.016	.026	.037	.050	.082	.14	.24	.45	1.0	1.9
O		↑	↓	.004	.007	.011	.016	.022	.035	.062	.11	.20	.37	.78	1.6
P		↓	.0013	.0025	.0041	.0060	.010	.015	.025	.041	.068	.15	.29	.62	1.2
Q		.00031	.00060	.0010	.0019	.0031	.0047	.0084	.014	.024	.045	.10	.21	.48	1.1

TABLE 5b

Table of $(t/\mu)x100$ Ratios for $\beta = 3/4$ for the 105B Plans

Sample Size Code Letter	Acceptable Quality Level (AQL) p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	(t/μ) x 100	$67x10^{-5}$	$21x10^{-4}$	$47x10^{-4}$	$84x10^{-4}$	$14x10^{-3}$	$29x10^{-3}$	$53x10^{-3}$	.10	.18	.31	.62	1.18	2.29	4.18
	$(t/\mu)x100$ at LTPD Quality Level [P(A) = .10]														
A															101
B														59	
C													30		
D												19			44
E											12			26	43
F										6.8			15	24	34
G									3.5			7.2	11	16	27
H								2.2			4.5	7.0	9.7	16	23
I							1.4			2.8	4.3	6.0	7.6	12	18
J						.80			1.6	2.5	3.4	4.3	6.4	9.8	16
K					.48			.99	1.5	2.0	2.6	3.8	5.0	8.2	14
L				.32			.65	.99	1.3	1.7	2.1	3.3	4.8	7.8	12
M			.19			.38	.58	.79	1.0	1.2	1.9	2.7	4.4	6.7	10
N		.13			.26	.38	.53	.67	.83	1.1	1.6	2.4	3.6	6.4	9.4
O				.15	.23	.30	.39	.47	.66	.95	1.4	2.1	3.2	5.2	8.5
P			.076	.11	.15	.20	.28	.38	.53	.75	1.0	1.7	2.7	4.5	7.2
Q		.030	.045	.065	.097	.13	.17	.25	.36	.51	.78	1.3	2.1	3.8	6.5

TABLE 5c

Table of (t/μ) x 100 Ratios for β = 1 for the 105B Plans

Sample Size Code Letter		Acceptable Quality Level (AQL)													
	p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	(t/μ) x 100	.015	.035	.065	.10	.15	.25	.40	.65	1.01	1.51	2.53	4.08	6.72	10.54
A	(t/μ) x 100 at LTPD Quality Level [P(A) = .10]														115
B														77	
C													46		
D												33			61
E											23			41	60
F										15			27	38	50
G									9.2			16	22	28	42
H								6.5			11	16	20	28	38
I							4.6			7.8	11	14	17	23	31
J						3.1			5.3	7.2	9.1	11	15	20	29
K					2.1			3.6	4.9	6.2	7.4	9.9	12	18	26
L				1.5			2.6	3.6	4.5	5.4	6.3	9.0	12	17	24
M			1.0			1.8	2.4	3.0	3.6	4.2	6.0	7.6	11	15	21
N		78			1.3	1.8	2.3	2.7	3.1	4.0	5.3	6.9	9.5	14	20
O				.86	1.2	1.5	1.8	2.1	2.7	3.5	4.5	6.2	8.7	13	18
P			.52	.72	.91	1.1	1.4	1.8	2.3	2.9	3.7	5.4	7.6	11	16
Q		.26	.35	.45	.63	.79	.97	1.3	1.7	2.2	3.0	4.5	6.5	9.9	15

TABLE 5d

Table of (t/μ)x100 Ratios for β = 1 1/3 for the 105B Plans

Sample Size Code Letter	Acceptable Quality Level (AQL) p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	(t/μ), x 100	.15	.28	.44	.61	.83	1.22	1.73	2.50	3.45	4.69	6.91	9.88	14.36	20.12
A	(t/μ)x100 at LTPD Quality Level [P(A) = .10]														121
B														89	
C													61		
D												47			75
E											36			56	74
F										26			41	53	65
G									18			28	35	43	57
H								14			21	27	33	43	53
I							11			16	20	25	28	36	46
J						7.9			12	15	18	20	26	33	43
K					6.0			8.9	11	13	15	19	23	30	39
L				4.8			7.0	8.9	11	12	14	18	22	29	37
M			3.5			5.3	6.6	7.8	9.0	10	13	16	21	26	34
N		2.8			4.2	5.3	6.3	7.2	8.0	9.6	12	15	18	25	32
O				3.1	3.9	4.7	5.3	5.9	7.1	8.7	11	14	17	23	30
P			2.1	2.7	3.2	3.6	4.5	5.2	6.3	7.5	9.0	12	16	21	28
Q		1.3	1.6	1.9	2.4	2.9	3.3	4.1	5.0	6.2	7.8	11	14	19	26

TABLE 5e

Table of (t/μ)x100 Ratios for β = 1 2/3 for the 105B Plans

Sample Size Code Letter	Acceptable Quality Level (AQL)														
	p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	(t/μ) x 100	.57	.94	1.37	1.78	2.26	3.08	4.07	5.46	7.08	9.07	12.33	16.42	22.15	29.01
A	(t/μ)x100 at LTPD Quality Level [P(A) = .10]														122
B														96	
C													70		
D												57			83
E											46			65	82
F										36			51	63	75
G									27			37	45	52	66
H								22			30	37	42	52	62
I							18			24	29	34	38	46	55
J						14			19	23	27	29	35	43	54
K					11			15	18	21	23	28	32	39	49
L				9.1			13	15	17	19	21	26	31	38	47
M			7.2			9.8	12	14	15	17	20	24	30	36	44
N		6.1			8.3	9.8	11	13	14	16	19	22	27	35	42
O				6.5	7.8	8.9	9.9	11	13	15	17	21	[illegible]	32	39
P			4.8	5.8	6.6	7.4	8.7	9.8	11	13	15	19	23	30	37
Q		3.1	3.8	4.4	5.3	6.1	6.9	8.2	9.6	12	14	17	21	28	35

TABLE 5f

Table of (t/μ)x100 Ratios for $\beta = 2$ for the 105B Plans

Sample Size Code Letter		Acceptable Quality Level (AQL)													
	p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	(t/μ) x 100	1.38	2.11	2.88	3.57	4.37	5.64	7.14	9.12	11.31	13.87	17.95	22.79	29.25	36.63
A	(t/μ)x100 at LTPD Quality Level [P(A) = .10]	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	121
B		↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	99	↕
C		↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	77	↕	↕
D		↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	65	↕	↕	88
E		↓	↓	↓	↓	↓	↓	↓	↓	↓	54	↕	↕	72	88
F		↓	↓	↓	↓	↓	↓	↓	↓	44	↕	↕	58	70	80
G		↓	↓	↓	↓	↓	↓	↓	34	↕	↕	45	52	60	73
H		↓	↓	↓	↓	↓	↓	29	↕	↕	38	44	50	60	69
I		↓	↓	↓	↓	↓	24	↕	↕	32	37	42	46	53	63
J		↓	↓	↓	↓	20	↕	↕	26	30	34	37	43	51	61
K		↓	↓	↓	16	↕	↕	21	25	28	31	35	39	47	57
L		↓	↓	14	↕	↕	18	21	24	26	28	33	39	47	55
M		↓	12	↕	↕	15	17	19	21	23	27	31	37	44	52
N		10	↕	↕	13	15	17	18	20	22	26	30	35	43	50
O		↕	↕	10	12	14	15	16	18	21	24	28	33	40	48
P		↕	8.1	9.5	11	12	13	15	17	19	22	26	31	38	45
Q		5.8	6.7	7.6	8.9	10	11	13	15	17	19	24	28	35	43

TABLE 5a

Table of $(t/\mu)x100$ Ratios for $\beta = 3\ 1/3$ for the 105B Plans

Sample Size Code Letter	Acceptable Quality Level (AQL)														
	p'(%)	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	(t/μ) x 100	7.34	10.24	12.32	14.03	15.84	18.47	21.27	24.62	28.03	31.68	36.98	42.69	49.57	56.73
A	$(t/\mu)x100$ at LTPD Quality Level [P(A) = .10]														116
B														103	
C													88		
D												79			96
E											72			85	96
F										63			75	83	90
G									54			63	70	76	85
H								49			57	63	68	76	83
I							44			52	57	61	65	71	78
J						39			46	50	54	57	62	68	76
K					35			41	45	48	51	55	59	65	73
L				32			37	41	44	46	48	54	58	65	72
M			28			33	36	39	41	43	47	51	57	63	69
N		26			30	33	35	38	39	42	46	50	54	62	67
O				27	29	31	33	35	38	41	44	48	53	59	65
P			23	25	27	29	31	33	35	38	41	46	51	57	64
Q		19	21	22	24	26	28	30	33	35	39	44	49	55	62

TABLE 5

Table of $(t/\mu) \times 100$ Ratios at the Acceptable Quality Level (AQL) for the 105B Plans

β	Acceptable Quality Level - p' (%)													
	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
1/3	$56x10^{-12}$	$72x10^{-11}$	$46x10^{-10}$	$17x10^{-9}$	$56x10^{-9}$	$26x10^{-8}$	$11x10^{-7}$	$46x10^{-7}$	$17x10^{-6}$	$59x10^{-6}$	$27x10^{-5}$	$11x10^{-4}$	$51x10^{-4}$	$19x10^{-3}$
1/2	$11x10^{-7}$	$51x10^{-7}$	$21x10^{-6}$	$50x10^{-6}$	$11x10^{-5}$	$31x10^{-5}$	$80x10^{-5}$	$21x10^{-4}$	$51x10^{-4}$	$11x10^{-3}$	$32x10^{-3}$	$83x10^{-3}$	.23	.56
3/4	$67x10^{-5}$	$21x10^{-4}$	$47x10^{-4}$	$84x10^{-4}$	$14x10^{-3}$	$29x10^{-3}$	$53x10^{-3}$	.10	.18	.31	.62	1.18	2.29	4.18
1	$15x10^{-3}$	$35x10^{-3}$	$65x10^{-3}$	.10	.15	.25	.40	.65	1.01	1.51	2.53	4.08	6.72	10.54
1-1/8	$41x10^{-3}$	$88x10^{-3}$	.15	.22	.32	.50	.76	1.18	1.73	2.49	3.94	6.02	9.37	13.98
1-1/4	$94x10^{-3}$	.18	.30	.43	.59	.89	1.30	1.92	2.71	3.75	5.67	8.31	12.38	17.74
1-1/3	.15	.28	.44	.61	.83	1.22	1.73	2.50	3.45	4.69	6.91	9.88	14.36	20.12
1-1/2	.31	.55	.83	1.11	1.45	2.04	2.80	3.87	5.16	6.77	9.55	13.13	18.31	24.71
1-2/3	.57	.94	1.37	1.78	2.26	3.08	4.07	5.46	7.08	9.07	12.33	16.42	22.15	29.01
2	1.38	2.11	2.88	3.57	4.37	5.64	7.14	9.12	11.31	13.87	17.95	22.79	29.25	36.63
2-1/2	3.32	4.68	5.98	7.11	8.36	10.27	12.39	15.06	17.90	21.08	25.90	31.35	38.28	45.82
3-1/3	7.94	10.24	12.32	14.03	15.84	18.47	21.27	24.62	28.03	31.68	36.98	42.69	49.57	56.73
4	12.21	15.10	17.62	19.62	21.71	24.68	27.76	31.35	34.93	38.68	44.01	49.59	56.18	62.85
5	18.72	22.17	25.10	27.36	29.67	32.87	36.12	39.81	43.40	47.09	52.21	57.45	63.46	69.44

TABLE 7

Single Sample Sizes and Acceptance Numbers for the 105B Plans

Sample size code letter	Sample size n	Acceptance Number - c													
		Acceptable Quality Level (AQL)													
		0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
A	2													↓	0↑
B	3												↓	0↑	
C	5											↓	0↑		↓
D	7										↓	0		↓	1
E	10									↓	0↑	↑	↓	1	2
F	15								↓	0↑		↓	1	2	3
G	25							↓	0↑		↓	1	2	3	5
H	35						↓	0↑		↓	1	2	3	5	7
I	50					↓	0↑		↓	1	2	3	4	6	9
J	75				↓	0		↓	1	2	3	4	6	9	13
K	110			↓	0	↑	↓	1	2	3	4	6	8	12	18
L	150		↓	0↑	↑	↓	1	2	3	4	5	8	11	17	24
M	225	↓	0		↓	1	2	3	4	5	8	11	17	24	34
N	300	0	↑	↓	1	2	3	4	5	7	10	14	20	32	44
O	450	↑	↓	1	2	3	4	5	7	10	14	20	29	43	62
P	750	↓	1	2	3	4	6	8	11	15	20	31	45	68	98
Q	1500	1	2	3	5	7	9	13	18	25	35	56	81	124	184

APPENDIX A

The Weibull Distribution as a Failure Distribution

This appendix describes the Weibull distribution which is used as a mathematical model for the procedures described in this report. The Weibull distribution is a member of the general failure distribution which can be derived as follows:

Let N = the initial number of items under life testing.

$S(x)$ = the number of items still surviving at time x.

$R(x)$ = $S(x)/N$ = the proportion of items still surviving at time x.

$F(x)$ = $1 - R(x)$ = the proportion of failures prior to time x.

$Z(x)$ = the instantaneous rate of failure at time x.

γ = An assumed time, prior to which no failure will occur.

The introduction of γ, some finite time, means when $x \leq \gamma$; $S(x) = N$, $R(x) = 1$, $F(x) = Z(x) = 0$ which is a boundary condition. A negative value for γ means that the items could fail prior to life-testing, i.e., the items could fail in storage, a common phenomenon for batteries, electrolytic capacitors, electron tubes and the like.

The following simple differential equation equates the decrement of items surviving in some small increment of time dx with the product of S(x) and Z(x). The negative sign indicates the decrement.

$$dS(x)/dx = -Z(x) \cdot S(x) \tag{A 1}$$

Divide both sides by n and transpose,

$$dR(x)/R(x) = -Z(x)\, dx \tag{A 2}$$

Now integrate both sides over the interval (γ, x),

$$\ln R(y) \Big|_{\gamma}^{x} = - \int_{\gamma}^{x} Z(y)\, dy + C \tag{A 3}$$

The integration constant C is zero, because of the boundary condition of $R(x) = 1$ and $Z(x) = 0$, for $x \leq \gamma$

Hence,

$$\ln R(x) = - \int_{\gamma}^{x} Z(y)\, dy \tag{A 4}$$

which can be rewritten as

$$F(x) = 1 - R(x) = 1 - \exp\left[-\int_{\gamma}^{x} Z(y)dy \right] = 1 - \exp\left[-M(x) \right] \text{ for } x \geq \gamma,$$

$$= 0\,, \quad \text{for} \quad x < \gamma\,. \tag{A 5}$$

This is the form that a failure distribution must take. Since $Z(x)$ is non-negative for all x, in order for $F(x)$ to have meaning, $M(x)$ must be non-decreasing in x. Although there are still infinitely many choices of $M(x)$, this form of distribution rules out many other distributions, for example the normal distribution, as a failure distribution.

If the instantaneous failure rate, $Z(x)$ is a constant over time say equal to λ, then $M(x) = \int_{\gamma}^{x} \lambda\, dy = \lambda(x-\gamma)$, for $x > \gamma$, and $M(x) = 0$ for $x \leq \gamma$. The resulting distribution is the well-known 2-parameter exponential distribution namely for $\lambda = 1/\theta$,

$$F(x) = 1 - \exp\left[-(x-\gamma)/\theta \right]\,, \quad \text{for } x \geq \gamma,\ \theta > 0,$$

$$= 0, \quad \text{otherwise.} \tag{A 6}$$

However, if the instantaneous failure rate changes with time in such a way that $M(x) = \frac{(x-\gamma)^{\beta}}{\alpha}$ for $x > \gamma$ and $M(x) = 0$ for $x \leq \gamma$, the resulting distribution will be the following:

$$F(x) = 1 - \exp\left[-(x-\gamma)^{\beta}/\alpha \right]\,, \text{ for } x \geq \gamma,\ \alpha,\ \beta > 0,$$

$$= 0, \quad \text{otherwise.} \tag{A 7}$$

This form of the failure distribution is known as the Weibull distribution. The three parameters of the Weibull distribution are,

- α - the "scale" parameter
- β - the shape parameter
- γ - the location parameter

Often the Weibull distribution is written as,

$$F(x) = 1 - \exp\left[-\left(\frac{x-\gamma}{\eta}\right)^{\beta} \right]\,, \text{ for } x \geq \gamma,\ \eta,\ \beta > 0\,,$$

$$= 0, \quad \text{otherwise.} \tag{A 8}$$

Then $\eta = \alpha^{1/\beta} = \alpha^{b}$, is the true scale parameter (η is also known as the characteristic life). Note that for the special case when $\beta = 1$, the Weibull distribution (Eq. A7 or A8) reduces to the 2-parameter exponential distribution (Eq. A6).

The first derivative of Eq. A8 with respect to x gives the Weibull density function,

$$f(x) = \frac{\beta}{\eta} \left(\frac{x-\gamma}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right], \text{ for } x \geq \gamma,\ \eta,\ \beta > 0,$$

$$= 0, \qquad \text{otherwise.} \qquad \text{(A 9)}$$

The instantaneous failure rate, $Z(x)$ for the Weibull case is $d\,M(x)/dx = \beta(x-\gamma)^{\beta-1}/\eta^{\beta}$ which is a decreasing (increasing) function in $(x-\gamma)$ if $\beta < 1$ ($\beta > 1$) and a constant equal to $1/\eta$ if $\beta = 1$, i.e., the exponential case. A plot for $f(x)$ vs $(x-\gamma)$ is shown in Figure 1 for $\eta = 1$ and for several values of β, the Weibull shape parameter.

The variable X may be standardized by introducing the relationship $Y = (X-\gamma)/\eta$. Then the Weibull cumulative distribution function (c.d.f.) and the corresponding probability density function (p.d.f.) of Y are the following:

$$G(y) = 1 - \exp\left[-y^{\beta} \right], \text{ for } y > 0,\ \beta > 0,$$

$$= 0, \qquad \text{otherwise.} \qquad \text{(A 10)}$$

$$g(y) = \beta\, y^{\beta-1} \exp\left[-y^{\beta} \right], \text{ for } y > 0,\ \beta > 0,$$

$$= 0, \qquad \text{otherwise.} \qquad \text{(A 11)}$$

These are known as the standard Weibull distributions. Since Figure 1 shows $f(x)$ vs $(x-\gamma)$ for $\eta = 1$, the figure also represents $g(y)$ for the same set of β values.

In order to find the moments of X it is sufficient to calculate the moments of Y. The k^{th} moment of Y about zero is

$$EY^{k} = \int_0^{\infty} y^{k}\, \beta\, y^{\beta-1}\, e^{-y^{\beta}}\, dy = \int_0^{\infty} z^{k/\beta} e^{-z}\, dz = \Gamma\left(\frac{k}{\beta} + 1\right) \qquad \text{(A 12)}$$

From this expression all moments of X may be found. The following are the mean, standard deviation and skewness of X, for $b = 1/\beta$,

$$\mu = \gamma + \eta\,\Gamma(b+1) \qquad \text{(A 13)}$$

$$\sigma = \eta\,[\,\Gamma(2b+1) - \Gamma^2(b+1)\,]^{\frac{1}{2}} \qquad \text{(A 14)}$$

$$\alpha_3 = [\,\Gamma(3b+1) - 3\,\Gamma(2b+1)\,\Gamma(b+1) + 2\Gamma^3(b+1)\,]/\sigma^3 \qquad \text{(A 15)}$$

These relationships show that the standard deviations, σ, and α_3, are free of γ, and the mean, μ, contains a linear term of γ.

In the case for which γ is known, the variable $(X-\gamma)$ will behave in accordance with a Weibull distribution of the form given by Eq. A 16 which corresponds to Eq. A 8 with the value of γ equal to zero. Hence, for this report where γ is assumed to be known, only the following expressions Eqs. A 16 & A 17 will be referred to as Weibull distributions:

$$\begin{aligned} F(x) &= 1 - \exp[\,-(x/\eta)^{\beta}\,]\,, \text{ for } x \geq 0,\ \eta,\ \beta > 0 \\ &= 0, \quad \text{otherwise.} \end{aligned} \qquad \text{(A 16)}$$

The corresponding p.d.f. is,

$$\begin{aligned} f(x) &= (\beta/\eta)\,(x/\eta)^{\beta-1}\ \exp[\,-(x/\eta)^{\beta}\,], \\ &\quad \text{for } x \geq 0,\ \eta,\ \beta > 0 \\ &= 0, \quad \text{otherwise.} \end{aligned} \qquad \text{(A 17)}$$

Figure 1 referred to previously, shows Eq. A 17 precisely for the various β values indicated.

APPENDIX B

Graphical Estimation of Weibull Parameters

In this report, both the shape parameter, β, and the location parameter, γ, are assumed to be known. These assumptions are made in view of empirical evidence from experience with field and laboratory failure data that indicates for some components that these two parameters tend to be constants over a wide range of applications. The approximate magnitude of these two parameters must be estimated from prior data in order to use for acceptance purposes the sampling plans reported here. This appendix outlines two simple graphical methods for estimating not only these two parameters but also the scale parameter η . Since the two methods to be described will involve plotting the lifelength data on probability papers, each of the two methods also provide a graphical test of fit to warn against the use of Weibull sampling plans if the fit is "poor". The two methods I & II of estimation are as follows:

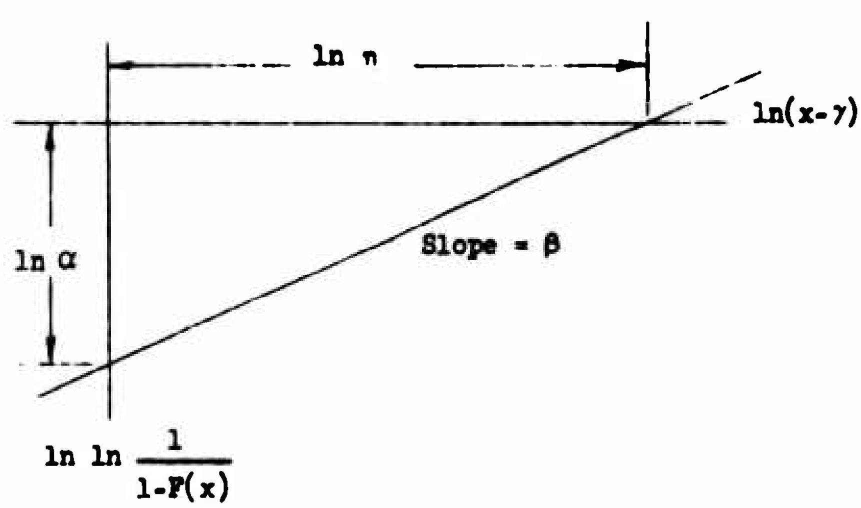
Method I

This method is based on the fact that Eq. A8 can be written as the following upon taking the logarithm twice:

$$\ln \ln \frac{1}{1-F(x)} = -\beta \ln \eta + \beta \ln (x-\gamma)$$

$$= -\ln \alpha + \beta \ln (x-\gamma) \qquad (B\ 1)$$

Hence on a graph paper with ln versus ln - ln coordinates the Weibull c.d.f. will appear as a straight line. The following sketch depicts such a straight line.



The Weibull probability paper (Figure 4) has four scales at its borders, two horizontal and two vertical. The upper and right scales are called the principal scales. The principle scales are linear because they are for the plotting of $\ln(x-\gamma)$ versus $\ln \ln \frac{1}{1-F(x)}$ which are linearly related according to Eq. B1. The lower and left scales, the auxilliary scales, are non-linear being calibrated for direct plotting of raw data. The two additional vertical scales along the righthand border of the paper labeled μ/η and σ/η are calibrated for β up to 7.0 in accordance with Eqs. A13 and A14, i.e., $\mu/\eta = \Gamma(b+1)$ ignoring γ and $\sigma/\eta = [\Gamma(2b+1) - \Gamma^2(b+1)]^{\frac{1}{2}}$. These scales facilitate calculation of μ and σ as will be illustrated later. The additional scales that appear at the top of the graph paper are for small values of β (0.02 to 1.5).

Consider the following hypothetical lifelength data (in units of 100 hrs.) consisting of ten ordered observations, $x_1 \leq x_2 \leq \ldots \leq x_{10}$, for which we estimate $F(x_i)$ by the so-called plotting positions: $i/(10+1)$, for $i = 1,2,\ldots,10$. (See the last paragraph of this appendix for other plotting conventions.) The observations are = 27.5, 31, 34, 38, 41, 44, 47, 51, 57 and 64. The corresponding plotting positions are = .09, .18, .27, .36, .46, .55, .64, .73, .82 and .91. These ten points plotted on the Weibull probability paper give Curve A (Fig. 4) which upon extending toward the bottom scale gives $\hat{\gamma} = 15$. If, however, the smallest observation, $x_1 = 27.5$, were chosen as an estimate of γ, the data after subtracting 27.5 from each observation will be = 0, 3.5, 6.5, 10.5, 13.5, 16.5, 19.5, 23.5, 29.5 and 36.5. With plotting positions remaining unchanged, the Weibull plot will then be Curve B. The fact that these two curves (A and B) have opposite curvatures indicates that the true location parameter γ lies somewhere between 15 and 27.5. Upon several trials, $\hat{\gamma} = 20$ (or thereabouts) gives the adjusted observations as 7.5, 11, 14, 18, 21, 24, 27, 31, 37 and 44. Using the same plotting positions as before, these adjusted observations give Curve C which is approximately linear. Curve C gives $\ln \hat{\alpha} = 5.96$, $\hat{\alpha} = 390$ (x in hundreds of hours), $\hat{\beta} = 1.85$, $\hat{\eta} = 27.5 \times 100$ hours. Some other life quality measures may now be estimated by a few simple calculations, thus (in hours), (Refer to Figure 4),

Mean = 2000 + .888(2750) = 4442 (See Eq. A13)

Std. dev. = .5(2750) = 1375 (See Eq. A 14)

Reliability function at 2300 or R(2300 - 2000)

= R(300) = 1 - .019) = .981

Reliable life at 90% = 2000 + 780 = 2780

Reliable life at 95% = 2000 + 530 = 2530

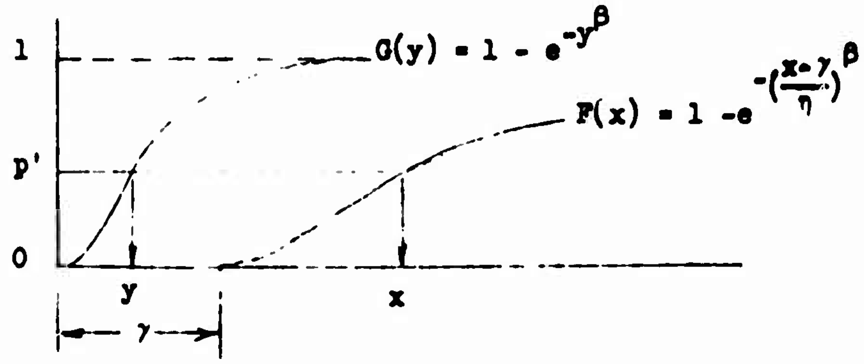
Median life = 2000 + 2230 = 4230

Initial failure rate per 100 hrs. = .26%.

Since $(x-\hat{\gamma})$ instead of x are plotted, the initial failure rate referred to in the last entry is actually the failure rate of components at age = 2000 hrs. The Curves labeled $L(y_1)$ and $U(y_1)$ are lower and upper confidence bands on $F(x)$ with confidence coefficient = 80%. For calculations to determine these curves, readers are referred to Kao[21]

Method II

This method is based on the relationship $Y = (X-\gamma)/\eta$ introduced in obtaining Eq. A10 from Eq. A8 which is shown in the following sketch.



From this process of standardization, we see that for $0 < p' < 1$, $p' = F(x) = G(y)$. Therefore we may write,

$$y = G^{-1}[p'] = G^{-1}[F(x)] \tag{B 2}$$

But the relationship $Y = (x-\gamma)/\eta$ is the same as,

$$X = \eta y + \gamma \tag{B 3}$$

which is the equation of a straight line. Hence on a graph paper with the

Y-axis calibrated in accordance with Eq. B2, the Weibull c.d.f. will again be a straight line. As a matter of fact, the above argument holds for all distributions, Weibull or otherwise, where F and G are the original and the standardized c.d.f. respectively. Unfortunately, for the Weibull case, the standard c.d.f., G, and hence also its inverse function G^{-1}, depends on β. (This is not so for the normal distribution or the exponential distribution.) As a result, for each value of β, it is necessary to give a different calibration for y from Eq. B2. This is precisely what is done for this second kind of Weibull probability paper (Fig. 5). The y-scale calibration for values other than integer values of β may be interpolated between indicated integers. For β-values of 1.0 and 3.0 (Curve D & E), the reversal of curvatures was noted. A few trials with other values resulted in Curve F with $\hat{\beta} = 1.85$. For $\hat{\beta} = 1.85$ it was found that the plot of original data was approximately linear. Extending Curve F toward the bottom border gives $\hat{\gamma} = 2000$. The estimate for the scale parameter $\eta = 2750$ is obtained by taking the difference between the X-value corresponding to Y = 1 and $\hat{\gamma}$ which is the x-value corresponding to Y = 0. Note that $\hat{\eta}$ is the slope of the line labeled Curve F.

There are many plotting conventions for estimating $p_i' = F(x_i)$ in addition to $i/(n+1)$ used here. The following gives a listing of some more widely used ones:

(1) Sample c.d.f., i/n

(2) Symmetrical Sample c.d.f., $(i-\frac{1}{2})/n$

(3) Mean of c.d.f., $i/(n+1)$

(4) Mode of c.d.f., $(i-1)/(n-1)$

(5) Median of c.d.f., $H^{-1}(\frac{1}{2} \mid i, n-i+1)$

(6) C.d.f. of EX_i, $G(EY_i) = F(EX_i)$

(7) Corrected c.d.f, $(i-3/8)/(n+\frac{1}{4})$

The merits of some of these plotting conventions are discussed in Chernoff and Lieberman[22] for the normal distribution and in Kimball[23] for the normal and Type I extreme-value distribution as well as in the text by Blom[24] which concerns itself mainly with the properties of plotting convention (7) in the list above for underlying normal distributions.

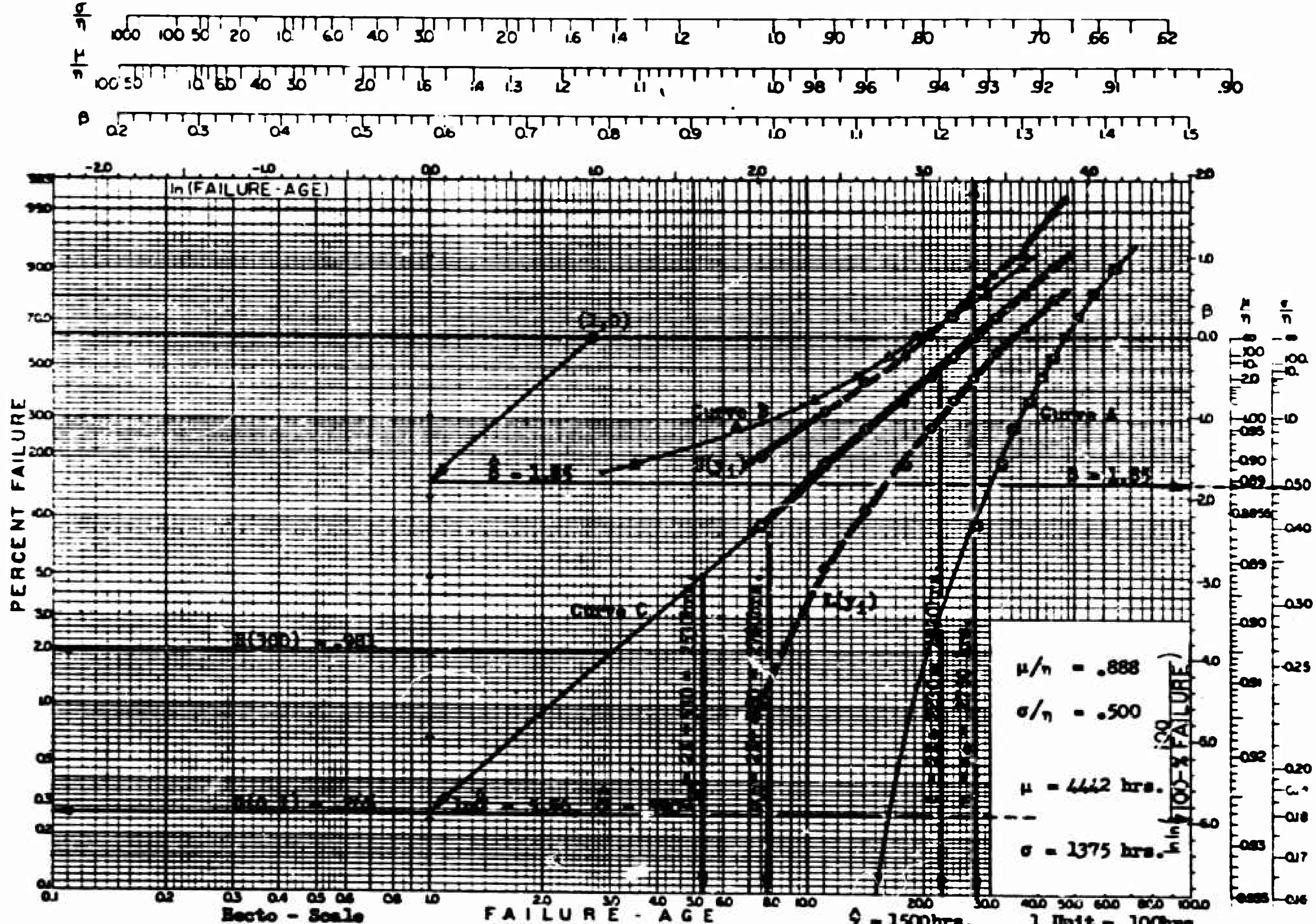
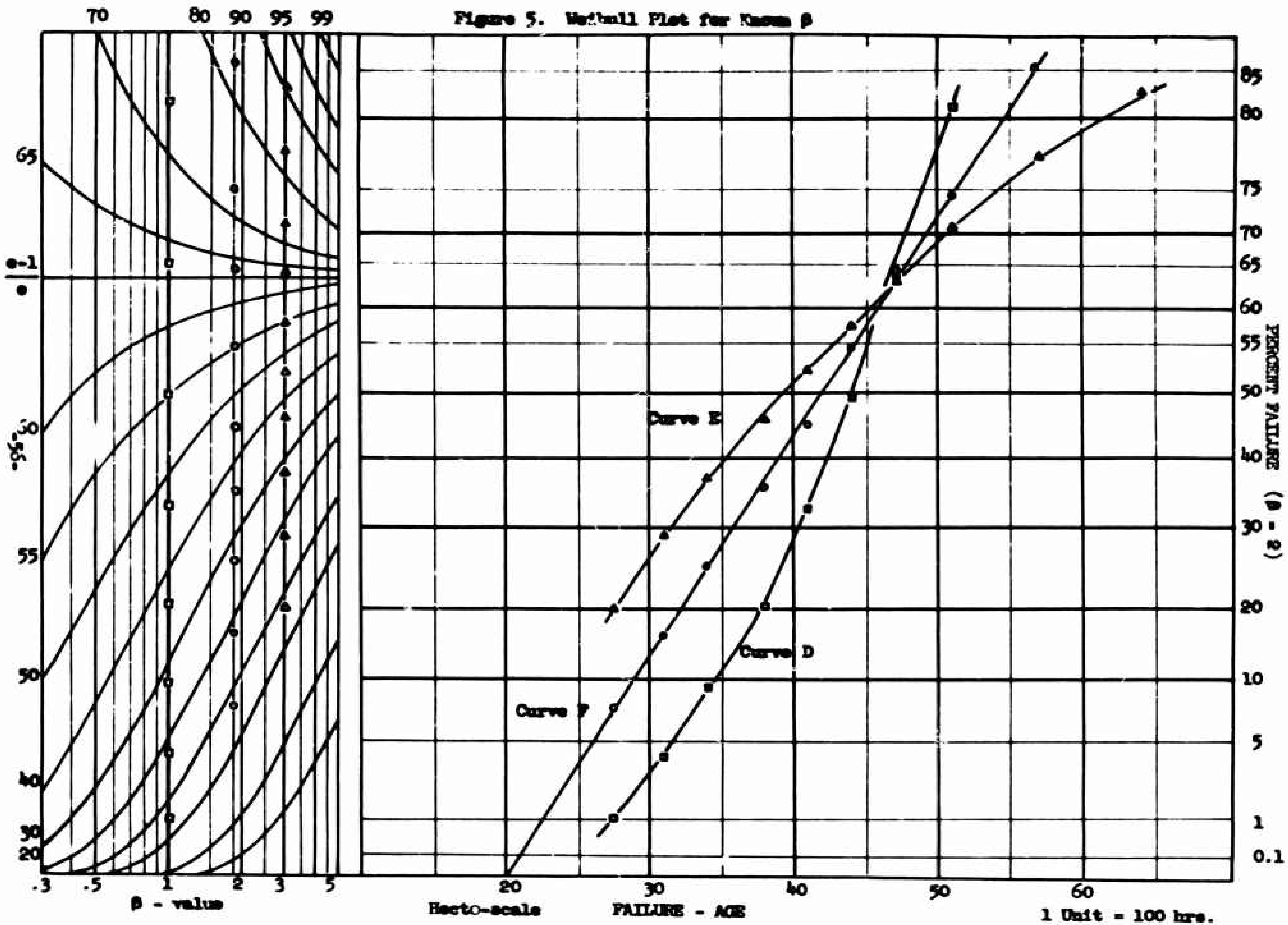


Figure 4 Weibull Probability Paper

Figure 5. Weibull Plot for Known β



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